

A Model for Counterparty Risk with Geometric Attenuation Effect and the Valuation of CDS

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January, 2007

Abstract: In this paper, a geometric function is introduced to reflect the attenuation speed of impact of one firm's default to its partner. If two firms are competitions (copartners), the default intensity of one firm will decrease (increase) abruptly when the other firm defaults. As time goes on, the impact will decrease gradually until extinct. In this model, the joint distribution and marginal distributions of default times are derived by employing the change of measure, so can we value the fair swap premium of a CDS.

Key words: Dependent default; Geometric attenuation function; Change of measure; Credit Default Swap(CDS)

2000 Mathematics Subject Classification: 62P05

Chinese Library Classification(GB/T13745): F830

1. Introduction

The rapid expansion in recent years of market for the credit derivatives had led to a growing interest in the valuation of these instruments including the credit default swaps(CDS).The reference issuers and the derivative issuers not only have default risk, but also correlate in some way. As remarked by Jarrow and Yu (2001), "an investigation of counterparty risk is incomplete without studying its impact on the pricing of credit derivatives". We can distinguish three different approaches to model default correlation in the literature of intensity credit risk modeling. The first approach to model default correlation makes use of copula functions. A copula is a function that links univariate marginal distributions to the joint multivariate distribution with auxiliary correlating variables. Li (2000) was probably the first to explicitly use the concept of copulas in the context of basket default derivatives pricing.

The second approach introduces correlation in firms' default intensities making them dependent on a set of common variables X_t and on a firm specific factor. These models have received the name of conditionally independent defaults (CID) models, because conditioned to the realization of the state variables X_t the firm's default intensities are independent as are the default times that they generate. for example Duffie and Singleton (1999) and Lando (1994, 1998).

*Project supported by the National Natural Science Foundation of China (No.70671069).

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The last approach to model default correlation, contagion models, relies on the works by Davis and Lo (1999) and Jarrow and Yu (2001). It is based on the idea of default contagion in which, when a firm defaults, the default intensities of related firms jump upwards. Leung and Kwok (2005) gave the analytic solution for the CDS premium for Jarrow and Yu (2001) model by employing the change of measure which was introduced in Collin-Dufresne (2004). But it is unrealistic for them to assume that one firm's default intensity keep a constant jump after the other firm defaults. In this paper, we introduce a geometric function to reflect the attenuation impact of one firm's default to other firms' default intensities. In our model, one firm's default will influence other firms' default intensities and the impact will decrease until extinct as time goes on. That is to say after a period of time, one firm's default intensity will depend the firm itself while the impact of other firm's default will disappear. The model is more realistic.

The paper is organized as follows. In section 2, we introduce a geometric function to reflect the attenuation impact of one firm's default to its partners, and give emphasis on the case when the two firms are competitions. The joint and marginal distributions of the two firms' default times are got by employing the change of measure which was introduced in Collin-Dufresne (2004). In section 3, we price the CDS premium using the conclusion of section 2 and get the analytic solution. The paper is ended with conclusion in the last section.

2. A Model for Dependent Default with Geometric Attenuation function

We consider an uncertain economy with a time horizon of T^* described by a filtered space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t=0}^{T^*}, P)$ satisfying $\mathcal{F} = \mathcal{F}_{T^*}$, where \mathbf{P} is the risk-neutral (equivalent martingale) measure in the sense of Harrison and Kreps (1981), that is, all security prices discounted by the risk-free interest rate r_t are martingale under \mathbf{P} .

In this section, we construct a two-firm model with default correlation. Suppose firm B and firm C have high direct linkage which are competitions or copartners. The default time of firm i ($i=B, C$) is denoted by τ^i which posses a strictly positive \mathcal{F}_t -predictable intensity process λ_t^i with right-continuous sample paths. If we define $N_t^i = I_{(\tau^i \leq t)}$ as the default indicator function which equals to 0 before firm i defaults and 1 otherwise.

Let $\mathcal{H}_t = \sigma(N_s^B, 0 \leq s \leq t) \vee \sigma(N_s^C, 0 \leq s \leq t)$, then we can get that

$$M_t^i = N_t^i - \int_0^{t \wedge \tau^i} \lambda_s^i ds \quad (1)$$

is an \mathcal{F}_t -local martingale and the conditional survival probability of firm i is given by

$$P(\tau^i > T | \mathcal{H}_t \vee \mathcal{F}_t) = I_{(\tau^i > t)} E[\exp(-\int_t^T \lambda_s^i ds) | \mathcal{F}_t]. \quad (2)$$

The default correlation between firm B and firm C is characterized by the correlated default intensities:

$$\lambda_t^B = b_0 + I_{(\tau^C \leq t)} \frac{b_1}{b_2(t - \tau^C) + 1}, \quad (3)$$

$$\lambda_t^C = c_0 + I_{(\tau^B \leq t)} \frac{c_1}{c_2(t - \tau^B) + 1}, \quad (4)$$

where b_0, c_0, b_2, c_2 are nonnegative real numbers, and b_1, c_1 are real numbers satisfying $b_0 + b_1 > 0, c_0 + c_1 > 0$. In this model, the default of firm C(B) will bring abrupt change to the default intensity of firm B(C). If B is one competition (copartner) of firm C, $b_1 < 0(> 0)$, and when firm C defaults, the default intensity λ_t^B jumps by the amount of $|b_1|$ from b_0 to $b_0 + b_1$. As time goes on, the impulsion effect will attenuate until extinct with geometric speed, that is to say, λ_t^B will come back to b_0 at last. This explanation is the same as C is one competition(copartner) of firm B. Parameters b_0 and c_0 are the firm-specific default intensity. Parameters b_1 and c_1 reflect the impact intensities of counterparty's default, and when $b_1 = 0$ and $c_1 = 0$, firm B and firm C are default-independent. Parameters b_2 and c_2 are non-negative real numbers, which reflect the attenuation speed. When $b_2 = 0$ and $c_2 = 0$, the model becomes the one in Jarrow and Yu (2001) and Yuen and Yue (2005).

To calculate the joint distribution of the two default time of firm B and firm C in $[0, T](T < T^*)$, we adopt the change of measure introduced by Collin-Dufresne (2004), defining a firm-specific probability measure P^i which puts zero probability on the pathes where default occurs prior to the maturity T. Specifically, the change of measure is defined by

$$Z_T^i \doteq \frac{dP^i}{dP} \Big|_{\sigma(\mathcal{H}_T \vee \mathcal{F}_T)} = I_{(\tau^i > T)} \exp\left(\int_0^T \lambda_s^i ds\right), \quad (5)$$

where P^i is a firm-specific(firm i) probability measure which is absolutely continuous with respect to P on the stochastic interval $[\tau^i, +\infty)$. To proceed the calculation under the measure P^i , we enlarge the filtration to $\mathcal{G}^i = (\mathcal{G}_t^i)_{t \geq 0}$ as the completion of $\sigma(\mathcal{H}_t \vee \mathcal{F}_t)_{t \geq 0}$ by the null set of the probability measure P^i . One can show that Z_T^i is a uniformly integrable P-martingale with respect to \mathcal{G}_T^i and almost surely strictly positive on $[0, \tau^i)$ and almost surely strictly equals to zero on $[\tau^i, +\infty)$ (see Collin-Dufresne (2004)).

Under the default risk structure specified in Eqs.(3) and (4), the survival probabilities of firm B and firm C are defined recursively through each other and this leads to the phenomenon of "looping default". Under the new measure P^B defined by Eq.(5), $\lambda_t^C = c_0$ for $t < \tau^C$, this effectively neglect the impact of firm B's default on the intensity of firm C, and looping default no longer exists. An analogous argument also holds under the measure P^C .

Proposition 1 When $-b_1 = b_2 = b > 0$ and $-c_1 = c_2 = c > 0$, the joint distribution of default times (τ^B, τ^C) with the default intensities defined by Eqs.(3) and(4) on $[0, T] \times [0, T]$ is found to be

$$P(\tau^B > t_1, \tau^C > t_2) = \begin{cases} c(t_2 - t_1 + \frac{1}{c} - \frac{1}{b_0})e^{-b_0 t_1 - c_0 t_2} + \frac{c}{b_0}e^{-(b_0 + c_0)t_2}, & \text{for } t_1 \leq t_2 \leq T \\ b(t_1 - t_2 + \frac{1}{b} - \frac{1}{c_0})e^{-b_0 t_1 - c_0 t_2} + \frac{b}{c_0}e^{-(b_0 + c_0)t_1}, & \text{for } t_2 < t_1 \leq T, \end{cases} \quad (6)$$

and the joint density is

$$f(t_1, t_2) = \begin{cases} cb_0 c_0 [(t_2 - t_1) + \frac{1}{c} - \frac{1}{c_0}]e^{-b_0 t_1 - c_0 t_2}, & \text{for } t_1 \leq t_2 \leq T \\ bb_0 c_0 [(t_1 - t_2) + \frac{1}{b} - \frac{1}{b_0}]e^{-b_0 t_1 - c_0 t_2}, & \text{for } t_2 < t_1 \leq T. \end{cases} \quad (7)$$

Proof:

If $t_2 \leq t_1 \leq T$,

$$(\tau^B > t_1, \tau^C > t_2) \in \mathcal{H}_{t_1} \subset \mathcal{G}_{t_1}^B. \quad (8)$$

So we can get

$$\begin{aligned} & P(\tau^B > t_1, \tau^C > t_2) \\ &= E^B[I_{(\tau^B > t_2)} \exp\{-\int_0^{t_1} (b_0 + I_{(\tau^C \leq t)}) \frac{-b_1}{(t-\tau^C+1)^{b_2}} dt\}] \\ &= E^B\left\{I_{(\tau^C > t_2)} e^{-b_0 t_1} \exp\{-b_1 I_{(\tau^C \leq t_1)} - \int_{\tau^C}^{t_1} \frac{1}{(t-\tau^C+1)^{b_2}} dt\}\right\} \end{aligned}$$

If $b_2 \neq 1$, the above equation equals to

$$\begin{aligned} &= e^{-b_0 t_1} E^B[I_{(t_2 < \tau^C \leq t_1)} \exp\{\frac{-b_1}{1-b_2} [(t_1 - \tau^C + 1)^{1-b_2} - 1] + I_{(\tau^C > t_1)}] \\ &= \exp\{-b_0 t_1 - \frac{b_1}{1-b_2}\} [\int_{t_2}^{t_1} c_0 e^{-c_0 t} [b(t_1 - t) + 1] bt + e^{-c_0 t_1}] \\ &= b(t_1 - t_2 + \frac{1}{b} - \frac{1}{c_0}) e^{-c_0 t_2 - b_0 t_1} + \frac{b}{c_0} e^{-(c_0 + b_0) t_1} \\ &= E^B\{I_{(\tau^C > t_2)} e^{-b_0 t_1} \exp\{I_{(\tau^C \leq t_1)} \ln[b(t_1 - \tau^C) + 1]\}\} \\ &= e^{-b_0 t_1} E^B[I_{(t_2 < \tau^C \leq t_1)} (b(t_1 - \tau^C) + 1) + I_{(\tau^C > t_1)}] \\ &= e^{-b_0 t_1} [\int_{t_2}^{t_1} c_0 e^{-c_0 t} [b(t_1 - t) + 1] bt + e^{-c_0 t_1}] \\ &= b(t_1 - t_2 + \frac{1}{b} - \frac{1}{c_0}) e^{-c_0 t_2 - b_0 t_1} + \frac{b}{c_0} e^{-(c_0 + b_0) t_1}, \end{aligned} \quad (9)$$

where E^C denotes the expectation under the measure P^C . The first equation follows from the definition of P^C , and the fourth from the fact that $\lambda_t^B = b_0$ for $t < t_2$ under P^C . By a similar argument for $t_2 \leq t_1 \leq T$, the joint distribution is given by

$$P(\tau^B > t_1, \tau^C > t_2) = b(t_1 - t_2 + \frac{1}{b} - \frac{1}{c_0}) e^{-b_0 t_1 - c_0 t_2} + \frac{b}{c_0} e^{-(b_0 + c_0) t_1}. \quad (10)$$

The differentiation of $P(\tau^B > t_1, \tau^C > t_2)$ with respect to t_1 and t_2 gives the joint density of the default times

$$f(t_1, t_2) = \frac{\partial^2 P(\tau^B > t_1, \tau^C > t_2)}{\partial t_1 \partial t_2} = \begin{cases} cb_0 c_0 [t_2 - t_1 + \frac{1}{c} - \frac{1}{c_0}] e^{-b_0 t_1 - c_0 t_2}, & \text{for } t_1 < t_2 \leq T \\ bb_0 c_0 [t_1 - t_2 + \frac{1}{b} - \frac{1}{b_0}] e^{-b_0 t_1 - c_0 t_2}, & \text{for } t_2 < t_1 \leq T, \end{cases} \quad (11)$$

The proof is completed. #

Remark 1 It is worth noting that if $cb_0 \neq bc_0$, then $f(t_1, t_2)$ is not continuous on the plane $t_1 = t_2$.

Corollary 1 Under the assumption of Proposition 1, if $b_1 = c_1 = 0$, then the joint distribution of default times (τ^B, τ^C) with the default intensity defined by Eq.(3)(4) on $[0, T] \times [0, T]$ is

$$P(\tau^B > t_1, \tau^C > t_2) = e^{-b_0 t_1 - c_0 t_2}, \quad (12)$$

in other words, when $b_1 = c_1 = 0$, (τ^B, τ^C) are default-independent on $[0, T] \times [0, T]$.

Proof We can get (11) by taking limit in Eq.(9) or Eq. (10) as $c \rightarrow 0^+$. #

Corollary 2 If $-b_1 = b_2 = b > 0, -c_1 = c_2 = c > 0$, the marginal distributions of τ^B, τ^C on $[0, T]$ are given by

$$P(\tau^B > t_1) = e^{-b_0 t_1} + \frac{b}{c_0} e^{-b_0 t_1} [e^{-c_0 t_1} - 1 + c_0 t_1], \quad t_1 \leq T \quad (13)$$

$$P(\tau^C > t_2) = e^{-c_0 t_2} + \frac{c}{b_0} e^{-c_0 t_2} [e^{-b_0 t_2} - 1 + b_0 t_2], \quad t_2 \leq T. \quad (14)$$

Proof We can get Eqs.(13) and (14) by taking $t_1 = 0$ and $t_2 = 0$ in Eqs.(9) and (10) respectively. $\#$

Remark 2 The first term in Eq.(13) denotes the firm-specific survival probability, and the second one denotes the increment of firm B's survival probability because of the default of firm C and the geometric attenuation speed. As the result of

$$e^{-c_0 t_1} - 1 + c_0 t_1 \leq \frac{1}{2} c_0^2 t_1^2,$$

the increment of firm B's survival probability satisfies

$$\frac{b}{c_0} e^{-b_0 t_1} [e^{-c_0 t_1} - 1 + c_0 t_1] \leq \frac{1}{2} b c_0 t_1^2 e^{-b_0 t_1} \leq \begin{cases} \frac{2c_0}{b_0} e^{-2} & \text{for } \frac{2}{b_0} \leq T, \\ \frac{1}{2} b c_0 T^2 e^{-b_0 T} & \text{for } \frac{2}{b_0} > T. \end{cases}$$

From the above inequation we can get that the increment of firm B's survival probability at time t will be no more than $\frac{1}{2} b c_0 t_1^2 e^{-b_0 t_1}$. An analogous argument also holds for firm C.

Section 3 CDS Valuation in the Model of Dependent Default with Geometric Attenuation function

In this section we use the conclusion of Section 2 to price the premium of a CDS. A CDS is a contract agreement between protection buyer and seller, in which the protection buyer pays periodically to the protection seller a fixed amount fee (swap premium pr spread) asking for a payment when the reference asset defaults. A institute can use a CDS to transfer, elude and hedge the credit risk of a risky asset (or basket of risky assets) from one party to the other. So a CDS is a very important instrument to manage credit risk.

Suppose interest rate r_t is a constant r . Assume that party A holds a corporate bond and faces the credit risk arising from default of the bond issuer (reference party C). To seek protection against such default risk, party A enters a CDS contract in which he agrees to make a stream of periodic premium payments, known as the swap premium to party B (CDS protection seller). In exchange, party B promises to compensate A (CDS protection buyer) for its loss in the event of default of the bond (reference asset). Without loss of generality, we take the notional to be \$ 1 and assume zero recovery under default. Firm B pays firm A after a settlement period δ when the reference asset c defaults. Furthermore in Leung and Kwok (2005), they conclude the expression for the swap premium has little dependence on the default intensity of the protection buyer, so we impose that during the entire contract, firm A doesn't default.

The default intensity processes of firm B and C are given by Eqs.(3) and (4) in special cases

$$\lambda_t^B = b_0 - I_{(\tau^C \leq t)} \frac{b}{b(t - \tau^C) + 1}, \quad (3)$$

$$\lambda_t^C = c_0 - I_{(\tau^B \leq t)} \frac{c}{c(t - \tau^B) + 1}. \quad (4)$$

Since it takes no cost to enter a CDS, the value of the swap premium $S(T)$ is determined by

$$\begin{aligned} & \sum_{i=1}^n E[e^{-rT_i} S(T) I_{(\tau^B \wedge \tau^C > T_i)}] + S(T) A(T) \\ = & E[e^{-r(\tau^C + \delta)} I_{(\tau^C \leq T)} I_{(\tau^B > \tau + \delta)}] \end{aligned} \quad (15)$$

where $\{T_1, \dots, T_n\}$ are the swap payment dates with $0 = T_0 < T_1 < \dots < T_n = T$, $T_i - T_{i-1} = \Delta T$, $T + \delta < T^*$, and δ is the length of settlement period. Here $\tau^C + \delta$ represents the settlement date at the end of the settlement period. The first term in Eq.(15) gives the present value of the sum of periodic swap payments(determined when either B or C defaults or at maturity) and $S(T)A(T)$ is the present value of the accrued swap premium for the fraction of period between τ^C and the last payment date. The present value of accrued swap premium is given by

$$S(T)A(T) = S(T) \sum_{i=1}^n E[e^{-r\tau^C} \frac{\tau^C - T_{i-1}}{\Delta T} I_{(T_{i-1} < \tau^C \leq T_i)} I_{(\tau^B > \tau^C)}]. \quad (16)$$

In the following, we will calculate all the expectations in Eq.(16). For simplicity we denote $\beta := b_0 + c_0 + r$.

It is easy to get from Eq.(6)

$$E[I_{(\tau^B \wedge \tau^C > T_i)}] = P(\tau^B > T_i, \tau^C > T_i) = e^{-(b_0 + c_0)T_i}, \quad (17)$$

So

$$\sum_{i=1}^n E[e^{-rT_i} S(T) I_{(\tau^B \wedge \tau^C > T_i)}] = S(T) \frac{e^{-\beta \Delta T} (1 - e^{-\beta T})}{1 - e^{-\beta \Delta T}}. \quad (18)$$

From Eq.(11) it can be found

$$\begin{aligned} & E[e^{-r(\tau^C + \delta)} I_{(\tau^C \leq T)} I_{(\tau^B > \tau^C + \delta)}] \\ = & \int_0^T \int_{t_2 + \delta}^{+\infty} e^{-r(t_2 + \delta)} b b_0 c_0 [t_1 - t_2 + \frac{1}{b} - \frac{1}{b_0}] e^{-b_0 t_1 - c_0 t_2} dt_1 dt_2 \\ = & \frac{b c_0}{\beta} (\frac{1}{b} + \delta) e^{-(r + b_0)\delta} [1 - e^{-\beta T}] \end{aligned} \quad (19)$$

and

$$\begin{aligned} & E[e^{-r\tau^C} \frac{\tau^C - T_{i-1}}{\Delta T} I_{(T_{i-1} < \tau^C \leq T_i)} I_{(\tau^B > \tau^C)}] \\ = & \int_{T_{i-1}}^{T_i} \int_{t_2}^{+\infty} e^{-r t_2} \frac{t_2 - T_{i-1}}{\Delta T} b b_0 c_0 [t_1 - t_2 + \frac{1}{b} - \frac{1}{b_0}] e^{-b_0 t_1 - c_0 t_2} dt_1 dt_2 \\ = & \frac{c_0}{\beta \Delta T} [T_{i-1} e^{-\beta T_{i-1}} - T_i e^{-\beta T_i} + (T_{i-1} - \frac{1}{\beta})(e^{-\beta T_i} - e^{-\beta T_{i-1}})]. \end{aligned} \quad (20)$$

So

$$\begin{aligned} S(T)A(T) &= S(T) \frac{c_0}{\beta \Delta T} [\sum_{i=1}^n (T_{i-1} e^{\beta T_{i-1}} - T_i e^{\beta T_i}) \\ & \quad + \sum_{i=1}^n T_{i-1} (e^{-\beta T_i} - e^{\beta T_i}) - \frac{1}{\beta} \sum_{i=1}^n (e^{-\beta T_i} - e^{-\beta T_{i-1}})] \\ &= S(T) \frac{c_0}{\beta \Delta T} \{-T e^{-\beta T} - \frac{1}{\beta} (e^{-\beta T} - 1) \\ & \quad + \frac{1}{(1 - e^{-\beta \Delta T})} [T e^{-\beta T} - \Delta T e^{-\beta \Delta T}] - (T - \Delta T) e^{-\beta(T + \Delta T)}\} \\ &= S(T) \frac{c_0}{\beta^2 \Delta T} \frac{1}{(1 - e^{-\beta \Delta T})} [1 - e^{-\beta \Delta T} - \beta \Delta T e^{-\beta \Delta T}]. \end{aligned}$$

That is to say

$$A(T) = \frac{c_0}{\beta^2 \Delta T} \frac{1}{(1 - e^{-\beta \Delta T})} [1 - e^{-\beta \Delta T} - \beta \Delta T e^{-\beta \Delta T}]. \quad (21)$$

Take Eqs.(18),(19) and (21) into (15), we can get

Proposition 2 Assume the default buyer doesn't default during the entire contract, and the default intensities of B (the protection seller) and C(Protection buyer) are given by Eq.(3) and (4), then the swap premium is given by

$$S(T) = \frac{bc_0}{\beta} \left(\frac{1}{b} + \delta \right) e^{-(r+b_0)\delta} [1 - e^{-\beta T}] \times \left[\frac{e^{-\beta \Delta T} (1 - e^{-\beta T})}{1 - e^{-\beta \Delta T}} + A(T) \right]^{-1}. \quad (22)$$

where $A(T)$ is given by Eq.(21).

Remark 2: Due to

$$1 - e^{-\beta \Delta T} - \beta \Delta T e^{-\beta \Delta T} \geq \frac{1}{2} \beta^2 \Delta T^2 e^{-\beta \Delta T},$$

the swap premium $S(T)$ is bounded by

$$S(T) \leq \frac{bc_0}{\beta} \left(\frac{1}{b} + \delta \right) e^{-(r+b_0)\delta} \frac{(1 - e^{-\beta T})(e^{\beta \Delta T} - 1)}{\frac{c_0 \Delta T}{2} + 1 - e^{-\beta T}}$$

Section 4. Conclusion

In this paper, a geometric function is introduced to reflect the attenuation speed of impact of one firm's default to its partner. If the two firms are competitions (copartners), the default intensity of one firm will decrease (increase) abruptly when the other firm defaults. As time goes on, the impulsion will decrease gradually until extinct. In this model, the joint distribution and marginal distributions of default times are derived by employing the change of measure, so can we value the fair swap premium of a CDS and get the analytic expression.

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