1 Workshop Information

1.1 Announcement

The workshop on non-linear partial differential equations invites some experts to share ideas and results on recent research in analysis of non-linear partial differential equations and their applications. The workshop will be held at Minhang campus of Shanghai Jiao Tong University on June 1-2, 2019.

1.2 Scientific Committee

- Congming Li (Shanghai Jiao Tong University)
- Wenxiong Chen (Yeshiva University)
- Chunqin Zhou (Shanghai Jiao Tong University)
- Fang Wang (Shanghai Jiao Tong University)
- Yutian Lei (Nanjing Normal University)
- Zhongxue Lü (Jiangsu Normal University)

1.3 Organizing Committee

- Jiankai Xu (Hunan Agricultural University)
- Genggeng Huang (Fudan University)
- Zhigang Wu (Donghua University)
- Yingshu Lü (Shanghai Jiao Tong University)

1.4 Venues

Room 630, No.6 Building, Science Buildings
Minhang Campus
Shanghai Jiao Tong University
800 Dongchuan Road

1.5 Hotel

Redding Mann Hotel
Address: No.178 Yongping Nan Road, Minhang District, Shanghai 200240, China
Telephone: 021-33882999/ 17302151313

1.6 Sponsors

School of Mathematical Sciences, Shanghai Jiao Tong University

1.7 Contact Us

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2 Schedule

June 1, Saturday: Room 630, No.6 Building, Science Buildings		
Time	Event (Speaker)	Title
Forenoon		
8:50-9:00	Opening ceremony	
Chair: Zhongxue Lü		
9:00-10:00	Wenxiong Chen	Maximum principles, sliding methods, and method of moving planes in unbounded regions
10:00-10:30	Leyun Wu	Some monotonicity results for some nonlocal problems in $\mathbb{R}^n$
10:30-11:00	Tea break	(Take a group photo)
Chair: Yutian Lei		
11:00-11:30	Fengping Yao	Lorentz estimates for a class of nonlinear parabolic systems
11:30-12:00	Xiaosen Han	Some Nonlinear Elliptic PDEs Arising in the Chern-Simons-Higgs Theory
12:00-14:30	Lunch	(At Campus Café)
Afternoon		
Chair: Wenxiong Chen		
14:30-15:30	Congming Li	Analysis of solutions to fractional Laplace equations
15:30-16:00	Zhigang Wu	Non-negative solutions to fractional Laplace equations with isolated singularity
16:00-16:20	Tea Break	
Chair: Chunqin Zhou		
16:20-16:50	Fang Wang	Eigenvalue comparison theorems for Poincare-Einstein manifolds
16:50-17:20	Chenkai Liu	On Dirichlet problem for the fractional Laplacian
17:20-17:50	Yingshu Lü	Maximum principles for Laplacian and fractional Laplacian with critical integrability
17:50-	Banquet	(At Liuyuan)

June 2, Sunday: Room 630, No.6 Building, Science Buildings		
Time	Event (Speaker)	Title
Forenoon		
Chair: Xiaohui Yu		
9:00-9:30	Chunqin Zhou	The exponential property of solutions bounded from below to fractional equations in unbounded cylinder
9:30-10:00	Ran Zhuo	Liouville type theorem for anti-symmetric equation involving fractional Laplacian on a upper half space
10:00-10:30	Jiankai Xu	On the integral equation with the axis-symmetric kernel
10:30-10:50	Tea break	
Chair: Jiankai Xu		
10:50-11:20	Genggeng Huang	Lioville theorem for degenerate elliptic equations
11:20-11:50	Xiaohui Yu	Liouville Type Theorems for Hartree and Hartree-Fock Equations
11:50-14:30	Lunch	(At Juxinge)
Afternoon		
Chair: Fang Wang		
14:30-15:00	Yan Li	Existence of positive solutions to nonlinear equations involving distinct fractional Laplacians
15:00-15:30	Shaodong Wang	A compactness theorem for boundary Yamabe problem in the scalar-flat case
15:35-16:05	Hao Xu	Qualitative analysis for semilinear elliptic systems
16:05-16:35	Meiqing Xu	The integrability estimate for nonnegative solutions of Lane-Emden system

3 Titles and Abstracts

Maximum principles, sliding methods, and method of moving planes in unbounded regions

Speaker: Wenxiong Chen (Yeshiva University)

Abstract: In this talk, we will summarized our recent results on maximum principles, sliding methods, and method of moving planes for nonlinear equations involving fractional Laplacians, fractional p-Laplacians, and other nonlocal nonlinear operators on unbounded domains. Our approaches are completely different from the traditional ones. Instead of estimating a sequence of equations , we evaluate the singular integrals defining the nonlocal operators along a sequence of approximate maximum points. This new method not only enable us to deal with nonlinear nonlocal operators, such as the fractional p-Laplacians, but also enable us to weaken the conditions on the domains and on the nonlinearities.

Some Nonlinear Elliptic PDEs Arising in the Chern-Simons-Higgs Theory

Speaker: Xiaosen Han (Henan University)

Abstract: In this talk, we review some results about the nonlinear elliptic pdes arising from Chern-Simons-Higgs(CSH) model. First we recall some elliptic pdes with related existence results arising from the classical Abelian Higgs and CSH models. Then we review the pdes appear in the non-Abelian CSH model and present some existence results. In particular, we report some theorems concerning the existence of doubly periodic solutions. At last, we mention some related open questions.

Lioville theorem for degenerate elliptic equations

Speaker: Genggeng Huang (Fudan University)

Abstract: In this talk, we apply the method of moving plane to the following high order degenerate elliptic equation,

$$(-A)^p u = u^\alpha \text{ in } \mathbb{R}^{n+1}_+, n \geq 1,$$

where the operator $A = y\partial_y^2 + a\partial_y + \Delta_x, a \geq 1$. We get a Liouville theorem for subcritical case and classify the solutions for the critical case. This is a joint work with Prof. Congming Li.

Analysis of solutions to fractional Laplace equations

Speaker: Congming Li (Shanghai Jiao Tong University)

Abstract: In this talk, we present some work on solutions to fractional Laplace equations including solutions with singularities. We present a short summary of our work on: some basic maximum principles, existence and uniqueness, symmetry and monotonicity, Bôcher type theorems singular solutions and related maximum principles. We will also present several basic lemmas unifying the treatments of Laplacian and fractional Laplacian.

Existence of positive solutions to nonlinear equations involving distinct fractional Laplacians

Speaker: Yan Li (Baylor University)

Abstract: Let $\Omega \in \mathbb{R}^n$ be a bounded domain with C^2 boundary. Consider

$$\begin{cases} (-\Delta)^{\alpha/2} u(x) = f(x, u(x), v(x)) & \text{if } x \in \Omega, \\ (-\Delta)^{\beta/2} v(x) = g(x, u(x), v(x)) & \text{if } x \in \Omega, \\ u(x) = v(x) = 0 & \text{if } x \in \Omega^c, \end{cases} \tag{3.1}$$

where $f, g \in C(\Omega, \mathbb{R}, \mathbb{R})$. When the solution (u, v) is a priori bounded, under some assumptions on $f(x, t, s)$ and $g(x, t, s)$ about their super-linearity with respect to t and s near zero and infinity, we prove that there exists at least one positive solution (u, v) using the topological degree theory. This is a joint work with Ran Zhuo.

On Dirichlet problem for the fractional Laplacian

Speaker: Chenkai Liu (Shanghai Jiao Tong University)

Abstract: In this talk, we consider the fractional Laplace (Poisson) equation in the distribution sense:

$$\begin{cases} (-\Delta)^s u = f & \text{in } \mathcal{D}'(B_1), \\ u = 0 & \text{in } B_1^c. \end{cases} \tag{3.2}$$

We give a nontrivial solution of (3.2) for $f = 0$.

To eliminate such non-uniqueness, we introduced two integrability conditions that can ensure the uniqueness. This is a joint work with Congming Li and Fanghua Lin.

Maximum principles for Laplacian and fractional Laplacian with critical integrability

Speaker: Yingshu Lü (Shanghai Jiao Tong University)

Abstract: In this talk, we discuss maximum principles for Laplacian and fractional Laplacian with critical integrability. We first consider

$$-\Delta u(x) + c(x)u(x) \geq 0 \quad \text{in } B_1,$$

where $c(x) \in L^p(B_1)$, $B_1 \subset \mathbf{R}^n$. As is known that $p = \frac{n}{2}$ is the critical case. We show that the maximum principle holds for $p \geq \frac{n}{2}$. On the other hand, the strong maximum principle requires $p > \frac{n}{2}$. In fact, we give a counterexample to illustrate that no matter how small $\|c\|_{L^p(B_1)}$ is, the strong maximum principle is false as $p = \frac{n}{2}$. Next, we investigate

$$-\Delta u(x) + \vec{b}(x) \cdot \nabla u(x) \geq 0 \quad \text{in } B_1,$$

where $\vec{b}(x) \in L^p(B_1)$. Here $p = n$ is the critical case. In contrast to the previous case, the maximum principle and strong maximum principle both hold for $p \geq n$. We also extend some of the results above to fractional Laplacian.

Eigenvalue comparison theorems for Poincare-Einstein manifolds

Speaker: Fang Wang (Shanghai Jiao Tong University)

Abstract: In this talk, I will give several eigenvalue comparison theorems associated to Poincare-Einstein manifolds, as well as characterisation of equalities, which leads to some rigidity theorems.

A compactness theorem for boundary Yamabe problem in the scalar-flat case

Speaker: Shaodong Wang (McGill University)

Abstract: In this talk, I will present some recent results on the compactness of the solutions to the Yamabe problem on manifolds with boundary. The compactness of Yamabe problem was introduced by Schoen in 1988. There have been a lot of works on this topic later on. This is a joint work with Sergio Almaraz and Olivaine Queiroz.

Some monotonicity results for some nonlocal problems in $\mathbb{R}^n$

Speaker: Leyun Wu (Northwestern Polytechnical University)

Abstract:In this talk, we consider some monotonicity results for solutions to the fractional Laplacian and the fractional p-Laplacian in $\mathbb{R}^n$ by the direct method of moving planes. Among which, we do not need any decay conditions at infinity on solutions. Also we are concerned with the one-dimensional symmetry results, which are closely related to the De Giorgi conjecture. This is a joint work with Mei Yu.

Non-negative solutions to fractional Laplace equations with isolated singularity

Speaker: Zhigang Wu (Donghua University)

Abstract: We present some results on nonnegative solutions to fractional Laplace equations with isolated singularity. First, we establish Bocher theorem for both Laplacian and fractional Laplacian. As a key application, we also develop some interesting maximum principles with isolated singularity. Besides, to prove the Bocher theorem still holds for general cases, we introduce several simple and useful lemmas, which enable us to unify the treatments for both Laplacian and fractional Laplacian. This is a joint work with Congming Li, Chenkai Liu and Hao Xu.

On the integral equation with the axis-symmetric kernel

Speaker: Jiankai Xu (Hunan Agricultural University)

Abstract: In this talk, we investigate some properties of positive solutions for a nonlinear integral equation with axis-symmetric kernel functions, which arises from weak type convolution-Young's inequality and the stationary magnetic compressible fluid stars. With the help of the method of moving plane and regularity lifting lemma, we show that all of positive solutions in certain functional spaces is symmetric and monotonic decreasing on the symmetric axis, and the integrable intervals of positive solutions are also obtained. Furthermore, by analyzing the decay rate of positive solutions in different directions, we show that the kind of integral equation does not admit radial solutions in a weighted functional space.

Qualitative analysis for semilinear elliptic systems

Speaker: Hao Xu (University of Colorado Boulder)

Abstract: In this talk, we study the existence of a nonlinear system

$$\begin{cases} -\Delta u = u^p v^q & \text{in } \mathbb{R}^n, \\ -\Delta v = u^s v^t & \text{in } \mathbb{R}^n, \\ u, v > 0. \end{cases} \tag{3.3}$$

The integrability estimate for nonnegative solutions of Lane-Emden system

Speaker: Meiqing Xu (Shanghai Jiao Tong University)

Abstract: The well-known Lane-Emden conjecture states that Lane-Emden system admits only trivial solution under subcritical case where $\frac{1}{p+1} + \frac{1}{q+1} > \frac{n-2}{n}$. Cheng, Huang, Li have put forward a sufficient and necessary condition about the decay rate on the exterior area. In this talk we will give the integrability estimate of nonnegative solution of Lane-Emden system on the exterior area. The main idea is to apply Kelvin transform, and then use bootstrap method to estimate the local integrability.

Lorentz estimates for a class of nonlinear parabolic systems

Speaker: Fengping Yao (Shanghai University)

Abstract: In this talk, we obtain the following local Lorentz estimates

$$B(|F|) \in L^{\gamma,q}_{loc} \Rightarrow B(|\nabla u|) \in L^{\gamma,q}_{loc} \quad \text{for any } \ \gamma > 1 \quad \text{and } \ 0 < q \leq \infty$$

of the weak solutions for a class of quasilinear parabolic systems

$$u_t - \operatorname{div}(a(|\nabla u|)\nabla u) = \operatorname{div}(a(|F|)F),$$

where $B(t) = \int_0^t \tau a(\tau)\, d\tau$ for $t \geq 0$.

Liouville Type Theorems for Hartree and Hartree-Fock Equations

Speaker: Xiaohui Yu (Shenzhen University)

Abstract: In this talk, I will introduce some Liouville theorems related to Hartree equation and Hartree-Fock equations. We use the moving plane method in an integral form to prove our results. These results can be used to interpret some physical phenomenons in quantum mechanics.

The exponential property of solutions bounded from below to fractional equations in unbounded cylinder

Speaker: Chunqin Zhou (Shanghai Jiao Tong University)

Abstract: We use a characterization of the fractional Laplacian as a Dirichlet to Neumann operator via an extension problem to the upper half space to study the structure of solutions bounded from below to fractional order linear equation

$$(-\Delta)^{\sigma}u(\mathbf{x}) = a(\mathbf{x})u(\mathbf{x}) + b(\mathbf{x}), \sigma \in (0,1)$$

in unbounded cylinder. Thus, we can study instead of the fractional order linear equation but an equivalent degenerate elliptic equation with a Neumann boundary condition in one dimension higher. In this way, we are able to apply local type arguments to obtain the desired results. After establishing the supremum estimates, the local boundary Hölder estimates and boundary Harnack inequality of the latter equation, we show that all solutions bounded from below are linear combinations of two special solutions (one end with exponential growth and exponential decay at the other end) with a bounded solution to the degenerate equation. Therefore, the conclusion holds for the fractional order linear equation.

Liouville type theorem for anti-symmetric equation involving fractional Laplacian on a upper half space

Speaker: Ran Zhuo (Huanghuai University)

Abstract: In this talk, we consider the following anti-symmetric equation involving fractional Laplacian:

$$\begin{cases} (-\Delta)^s u(x) = u^p(x), & x \in \mathbb{R}^n_+, \\ u(x', -x_n) = -u(x', x_n), & x = (x', x_n) \in \mathbb{R}^n, \end{cases} \tag{3.4}$$

here $x' = (x_1, x_2, \ldots, x_{n-1})$.

We first derived *maximum principle for anti-symmetric function involving the fractional Laplacian in* $\mathbb{R}^n_+$. Based on the above *maximum principle in* $\mathbb{R}^n_+$, we employ method of moving planes to obtain that the positive solutions only depend on x_n variable in the critical case and subcritical case. Moreover, we show that the positive solutions are monotone increasing with respect to x_n variable. Finally, we obtained non-existence of positive solutions for (3.4). Some new ideas are involved in the proof, which may hopefully be applied to many other problems.

4 List of Participants

Name	Affiliation
Wenxiong Chen	Yeshiva University
Xiaosen Han	Henan University
Genggeng Huang	Fudan University
Yutian Lei	Nanjing Normal University
Congming Li	Shanghai Jiao Tong University
Yan Li	Baylor University
Tianyu Liao	Shanghai Jiao Tong University
Chenkai Liu	Shanghai Jiao Tong University
Yingshu Lü	Shanghai Jiao Tong University
Zhongxue Lü	Jiangsu Normal University
Fang Wang	Shanghai Jiao Tong University
Shaodong Wang	McGill University
Leyun Wu	Northwestern Polytechnical University
Zhigang Wu	Donghua University
Hao Xu	University of Colorado Boulder
Jiankai Xu	Hunan Agricultural University
Meiqing Xu	Shanghai Jiao Tong University
Fengping Yao	Shanghai University
Xiaohui Yu	Shenzhen University
Chunqin Zhou	Shanghai Jiao Tong University
Ran Zhuo	Huanghuai University