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Structure identification of uncertain dynamical networks coupled with complex-variable chaotic systems

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Abstract: Topological structures and node dynamics of dynamical networks have important influence on their dynamical behaviours. In practical applications, not all of them can be well determined beforehand; therefore this study investigates the structure identification of an uncertain dynamical network coupled with complex-variable chaotic systems. Based on the Barbalat's lemma, corresponding network estimators are designed for identifying the unknown or uncertain topological structure and node dynamics. Notably, the node dynamics need not to be identical and the topological structure need not to be symmetric or irreducible. Furthermore, this method can also better monitor the switching topological structure of a dynamical network. Numerical simulations are provided to verify the correctness and effectiveness of the theoretical results.

Nomenclature

Throughout this paper, for any complex number (or complex vector) x, x^r and x^i denote its real and imaginary parts, respectively. $\bar{x}$ denotes the complex conjugate of x. $\|x\|$ denotes the norm of x, defined by $\|x\| = \sqrt{x^{\mathrm{T}}\bar{x}}$.

1 Introduction

Dynamical networks have been successfully used to model many large scale physical systems consisting of large number of interactive individuals [1, 2]. Synchronisation as a typical collective dynamical behaviour of networks has attracted increasing attention from researchers and engineers, and has many potential practical applications [3–10]. Therefore how to control a dynamical network to a desired synchronisation status is an interesting and challenging problem.

Recently, many kinds of control schemes have been adopted to design controllers for achieving synchronisation, such as feedback control, adaptive control, intermittent control, pinning control and so on. In particular, pinning control has been widely investigated in various network models in virtue of lower control cost and easier implementation. In [11], authors investigated pinning control of scale-free dynamical networks and shown that the specifically pinning of the most highly connected nodes requires a significantly smaller number of local controllers as compared to the randomly pinning scheme. In [12], authors studied three fundamental problems in pinning control of dynamical networks, that is, what type of pinning schemes may be chosen, what kind of controllers may be designed and how large the coupling strength should be used for synchronising a given network. In [13], authors investigated pinning synchronisation of both undirected and directed dynamical networks and proposed specifically pinning schemes to select pinned nodes by investigating the relationship among pinning synchronisation, network topology and the coupling strength.

Clearly, all pinning control discussions above are based on the assumption that all the parameters of these networks, including topological structure and local node dynamics, are known in advance. However, not all parameters can be well determined beforehand in practical applications, that is, there may exist unknown or uncertain parameters. For example, in many cellular networks, the interactions between the protein–DNA are sometimes unknown, which can be viewed as the topological structure of the cellular networks. However, the DNA interactions play a crucial role in cellular processes, so identifying effectively the interactions becomes more interesting and important. Therefore structure identification of uncertain dynamical networks becomes a crucial issue [14–20]. In [14], authors suggested for the first time a method for estimating the topology of a network based on the dynamical evolution supported on the network. In [18], authors investigated structure identification of dynamical networks with both topological structure and node dynamics. In [20], authors studied topology identification and adaptive synchronisation of uncertain complex networks with adaptive double scaling functions.

Complex-variable chaotic systems have been used to model physical systems involving complex variables [21–24]. In [25], authors investigated synchronisation of a network coupled with complex-variable chaotic systems via pinning control and intermittent control, in which the topological structure and node dynamics are known. For unknown complex-variable dynamical networks, how to identify the unknown or uncertain topological structure and node dynamics has not been studied yet. Thus, this paper considers structure identification of uncertain dynamical networks coupled with complex-variable chaotic systems. Based on the Barbalat's lemma, an adaptive control scheme is adopted to design corresponding network estimators for identifying the unknown systems. Notably, the local node dynamics need not to be identical and the topological structure need not to be symmetric or irreducible. Furthermore, from numerical simulations, one can find that this method can also better monitor the switching topological structures of dynamical networks.

This paper is organised as follows. Section 2 introduces some preliminaries and the network models. Section 3 considers the structure identification and designs corresponding network estimators. Section 4 provides several numerical simulations to verify the correctness and effectiveness of the derived results. Section 5 concludes the paper.

2 Model and preliminaries

Consider a network consisting of N individuals indexed by $k = 1, 2, \ldots, N$, described by complex-variable chaotic systems

$$\dot{x}_k(t) = f_k(x_k(t), \alpha_k) \tag{1}$$

or

$$\dot{x}_k(t) = g_k(x_k(t)) + F_k(x_k(t))\alpha_k \tag{2}$$

where $x_k(t) = (x_{k1}(t), x_{k2}(t), \ldots, x_{kn}(t))^{\mathrm{T}} \in \mathbb{C}^n$ is an n-dimensional complex vector with $x_{kl}(t) = x_{kl}^r(t) + jx_{kl}^i(t)$ and $j = \sqrt{-1}$; $f_k, g_k : \mathbb{C}^n \to \mathbb{C}^n$ are n-dimensional complex-valued vector functions; $\alpha_k \in \mathbb{R}^{p_k}$ is a p_k-dimensional system parameter vector of the kth individual; $F_k : \mathbb{C}^n \to \mathbb{C}^{n\times p_k}$ is an $n \times p_k$ matrix.

Then, the networked system is described by

$$\dot{x}_k(t) = f_k(x_k(t), \alpha_k) + \varepsilon \sum_{l=1}^{N} a_{kl} H x_l(t), \quad k = 1, 2, \ldots, N \tag{3}$$

where $\varepsilon > 0$ is the coupling strength, $H = \mathrm{diag}(h_1, \ldots, h_n) \in \mathbb{R}^{n\times n}$ is the inner coupling matrix, $A = (a_{kl}) \in \mathbb{R}^{N\times N}$ is the zero-row-sum outer coupling matrix determining the topology and coupling strength of the network, which are defined as: if node k is affected by node l $(l \neq k)$, then $a_{kl} \neq 0$; otherwise, $a_{kl} = 0$.

Assumption 1: Suppose that there exists a constant diagonal matrix $M = \mathrm{diag}(m_1, m_2, \ldots, m_n)$ such that the complex-valued vector functions $f_k(x)$ satisfies

$$\begin{aligned}(y - x)^{\mathrm{T}}\overline{(f_k(y, \alpha_k) - f_k(x, \alpha_k))} + \overline{(y - x)}^{\mathrm{T}}(f_k(y, \alpha_k) - f_k(x, \alpha_k)) \\ \le (y - x)^{\mathrm{T}} M \overline{(y - x)}\end{aligned}$$

for all $x, y \in \mathbb{C}^n$, $k = 1, 2, \ldots, N$.

Remark 1: It is easy to check that many typical complex-variable chaotic systems, such as the complex-variable Chen, Lü and Lorenz systems, all satisfy Assumption 1.

Assumption 2: Denote $F_k(y_k(t)) = (F_k^1(y_k(t)), \ldots, F_k^{p_k}(y_k(t)))$ with $F_k^l(y_k(t)) \in \mathbb{C}^n$ for $l = 1, \ldots, p_k$. Suppose that $\{\{F_k^l(y_k(t))\}_{l=1}^{p_k}, Hy_k(t)\}_{k=1}^N$ are linearly independent on the orbits $\{y_k(t)\}_{k=1}^N$ for $t > 0$.

Lemma 1 (Barbalat's lemma [26])*:* If $\Phi : R \to R^+$ is a uniformly positive function for $t \ge 0$ and if the limit of the integral $\lim_{t\to\infty}\int_0^t \Phi(\tau)\mathrm{d}\tau$ exists and is finite, then $\lim_{t\to\infty}\Phi(t) = 0$.

3 Structure identification of uncertain dynamical networks

In this section, structure identification of uncertain dynamical networks coupled with complex-variable chaotic systems are studied in two cases.

First, assume that the topology of network (3) is unknown and to be identified, that is, outer coupling A is unknown. Our objective here is to identify the unknown topology through designing proper network estimators and adaptive updating laws. We then consider the following network as the estimator

$$\begin{aligned}
\dot{y}_k(t) &= f_k(y_k(t), \alpha_k) + \varepsilon \sum_{l=1}^{N} \hat{a}_{kl}(t) H y_l(t) + u_k(t) \\
u_k(t) &= -d_k(t)(y_k(t) - x_k(t)) \\
\dot{d}_k(t) &= \delta_k (y_k(t) - x_k(t))^{\mathrm{T}}\overline{(y_k(t) - x_k(t))} \\
\dot{\hat{a}}_{kl}(t) &= -\theta_{kl}((y_k(t) - x_k(t))^{\mathrm{T}} H \overline{y_l(t)} \\
&\quad + y_l^{\mathrm{T}}(t) H \overline{(y_k(t) - x_k(t))})
\end{aligned} \tag{4}$$

where $y_k(t) = (y_{k1}(t), y_{k2}(t), \ldots, y_{kn}(t))^{\mathrm{T}} \in \mathbb{C}^n$, $k = 1, 2, \ldots, N$, $l = 1, 2, \ldots, N$, $\delta_k > 0$, $\theta_{kl} > 0$ are constants, $\hat{A} = (\hat{a}_{kl}) \in \mathbb{R}^{N\times N}$ is the estimated matrix of A.

Let the errors $e_k(t) = y_k(t) - x_k(t)$, $k = 1, 2, \ldots, N$. Then the error system is

$$\begin{aligned}
\dot{e}_k(t) &= f_k(y_k(t), \alpha_k) - f_k(x_k(t), \alpha_k) + \varepsilon \sum_{l=1}^{N} \hat{a}_{kl}(t) H y_l(t) \\
&\quad - \varepsilon \sum_{l=1}^{N} a_{kl} H x_l(t) + u_k(t) \\
&= f_k(y_k(t), \alpha_k) - f_k(x_k(t), \alpha_k) + \varepsilon \sum_{l=1}^{N} a_{kl} H e_l(t) \\
&\quad + \varepsilon \sum_{l=1}^{N} (\hat{a}_{kl}(t) - a_{kl}) H y_l(t) + u_k(t)
\end{aligned} \tag{5}$$

Theorem 1: Suppose that (A1) and (A2) hold. Then the unknown coupling matrix A of the uncertain dynamical network (3) can be identified by the estimated matrix $\hat{A}$ of the network estimator (4).

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Proof: Consider the following Lyapunov function

$$V(t) = \sum_{k=1}^{N} e_k^{\mathrm{T}}(t)\overline{e_k(t)} + \sum_{k=1}^{N} \frac{(d_k(t) - d_k^*)^2}{\delta_k} + \sum_{k=1}^{N}\sum_{k=1}^{N} \frac{\varepsilon(\hat{a}_{kl}(t) - a_{kl})^2}{2\theta_{kl}}$$

where d_k^* $(k = 1, 2, \ldots, N)$ are sufficiently large positive constants to be determined.

The derivative of $V(t)$ along the trajectories of (4) and (5) satisfies (see equation at the bottom of the page)

where $E(t) = (e_1^{\mathrm{T}}(t), e_2^{\mathrm{T}}(t), \ldots, e_N^{\mathrm{T}}(t))^{\mathrm{T}}$, I_n and I_N are n-dimensional and N-dimensional identity matrices, '$\otimes$' denotes the Kronecker product, and $D = \mathrm{diag}(d_1^*, d_2^*, \ldots, d_N^*)$.

Let $m^* = \max_{1\le p\le n}\{m_p\}$, $d^* = \min_{1\le k\le N}\{d_k^*\}$ and $\lambda_{\max}$ be the largest eigenvalue of $(A^{\mathrm{T}} + A) \otimes H$, then one can choose $d^* = m^* + \varepsilon\lambda_{\max} + 1$ such that

$$\dot{V}(t) \le -E^{\mathrm{T}}(t)\overline{E(t)} \tag{6}$$

Then one has

$$\lim_{t\to\infty}\int_0^t E^{\mathrm{T}}(s)\overline{E(s)}\,\mathrm{d}s \le -\lim_{t\to\infty}\int_0^t \dot{V}(s)\,\mathrm{d}s = V(0) - \lim_{t\to\infty} V(t)$$

and $\sup_{t\ge 0} V(t) \le V(0)$, that is, $V(t)$ is bounded. That is to say, $E(t) \in L^2$ and $E(t)$ is bounded, that is $E(t) \in L^\infty$. From error system (5), one has $\dot{E}(t)$ exists and is bounded for $t \in [0, \infty)$. According to Barbalat's lemma, one obtains $\lim_{t\to\infty} E(t) = 0$, which leads $\lim_{t\to\infty}\dot{E}(t) = 0$. Then, $y(t) = (y_1^{\mathrm{T}}(t), y_2^{\mathrm{T}}(t), \ldots, y_N^{\mathrm{T}}(t))^{\mathrm{T}}$ converges to the following largest invariant set

$$\mathcal{E} = \{y(t) : \sum_{l=1}^{N}(\hat{a}_{kl}(t) - a_{kl})Hy_l(t) = 0,\ k = 1, 2, \ldots, N\}, \quad \text{as } t \to \infty$$

From (A2), one has $\hat{a}_{kl}(t) \to a_{kl}$, that is, the unknown coupling matrix A can be identified when the synchronisation between networks (3) and (4) is achieved. □

Second, assume that the topology A and system parameter α_k of network (3) are unknown and to be identified. Consider the following network as estimator

$$\begin{aligned}
\dot{y}_k(t) &= g_k(y_k(t)) + F_k(y_k(t))\hat{\alpha}_k(t) + \varepsilon\sum_{l=1}^{N}\hat{a}_{kl}(t)Hy_l(t) + u_k(t)\\
u_k(t) &= -d_k(t)e_k(t), \quad \dot{d}_k(t) = \delta_k e_k^{\mathrm{T}}(t)\overline{e_k(t)}\\
\dot{\hat{\alpha}}_k(t) &= -\eta_k\left(\overline{F_k(y_k(t))}^{\mathrm{T}} e_k(t) + F_k^{\mathrm{T}}(y_k(t))\overline{e_k(t)}\right)\\
\dot{\hat{a}}_{kl}(t) &= -\theta_{kl}(e_k^{\mathrm{T}}(t)H\overline{y_l(t)} + y_l^{\mathrm{T}}(t)H\overline{e_k(t)})
\end{aligned} \tag{7}$$

where $k = 1, 2, \ldots, N$, $l = 1, 2, \ldots, N$, $\delta_k > 0$, $\eta_k > 0$ and $\theta_{kl} > 0$ are constants, $\hat{\alpha}_k(t)$ and $\hat{a}_{kl}(t)$ are the estimated values of α_k and a_{kl}.

Then, the error system can be written as

$$\begin{aligned}
\dot{e}_k(t) =& f_k(y_k(t), \alpha_k) - f_k(x_k(t), \alpha_k) + F_k(y_k(t))(\hat{\alpha}_k(t) - \alpha_k)\\
&+ \varepsilon\sum_{l=1}^{N} a_{kl}He_l(t) + \varepsilon\sum_{l=1}^{N}(\hat{a}_{kl}(t) - a_{kl})Hy_l(t) + u_k(t)
\end{aligned} \tag{8}$$

Theorem 2: Suppose that (A1) and (A2) hold. Then the unknown system parameters α_k and coupling matrix A of the uncertain dynamical network (3) can be identified by the estimated values $\hat{\alpha}_k$ and $\hat{A}$ of the network estimator (7).

Proof: Consider the following Lyapunov function

$$\begin{aligned}
V(t) =& \sum_{k=1}^{N} e_k^{\mathrm{T}}(t)\overline{e_k(t)} + \sum_{k=1}^{N} \frac{(d_k(t) - d_k^*)^2}{\delta_k}\\
&+ \sum_{k=1}^{N}\sum_{k=1}^{N} \frac{\varepsilon(\hat{a}_{kl}(t) - a_{kl})^2}{2\theta_{kl}}\\
&+ \sum_{k=1}^{N} \frac{(\hat{\alpha}_k(t) - \alpha_k)^{\mathrm{T}}(\hat{\alpha}_k(t) - \alpha_k)}{2\eta_k}
\end{aligned}$$

where d_k^* $(k = 1, 2, \ldots, N)$ are sufficiently large positive constants to be determined.

$$\begin{aligned}
\dot{V}(t) =& \sum_{k=1}^{N}\dot{e}_k^{\mathrm{T}}(t)\overline{e_k(t)} + \sum_{k=1}^{N} e_k^{\mathrm{T}}(t)\overline{\dot{e}_k(t)} + \sum_{k=1}^{N}\frac{2(d_k(t) - d_k^*)\dot{d}_k(t)}{\delta_k} + \sum_{k=1}^{N}\sum_{k=1}^{N}\frac{\varepsilon(\hat{a}_{kl}(t) - a_{kl})\dot{\hat{a}}_{kl}(t)}{\theta_{kl}}\\
=& \sum_{k=1}^{N}\left(\left(f(y_k(t), \alpha_k) - f(x_k(t), \alpha_k)\right)^{\mathrm{T}}\overline{e_k(t)} + e_k(t)^{\mathrm{T}}\overline{\left(f(y_k(t), \alpha_k) - f(x_k(t), \alpha_k)\right)}\right)\\
&+ \varepsilon\sum_{k=1}^{N}\sum_{l=1}^{N}\left(a_{kl}e_l^{\mathrm{T}}(t)H\overline{e_k(t)} + a_{kl}e_k^{\mathrm{T}}(t)H\overline{e_l(t)} + (\hat{a}_{kl}(t) - a_{kl})\left(y_l^{\mathrm{T}}(t)H\overline{e_k(t)} + e_k^{\mathrm{T}}(t)H\overline{y_l(t)}\right)\right)\\
&- 2\sum_{k=1}^{N} d_k(t)e_k^{\mathrm{T}}(t)\overline{e_k(t)} + 2\sum_{k=1}^{N}(d_k(t) - d_k^*)e_k^{\mathrm{T}}(t)\overline{e_k(t)} - \varepsilon\sum_{k=1}^{N}\sum_{l=1}^{N}(\hat{a}_{kl}(t) - a_{kl})\left(y_l^{\mathrm{T}}(t)H\overline{e_k(t)} + e_k^{\mathrm{T}}(t)H\overline{y_l(t)}\right)\\
\le& \sum_{k=1}^{N} e_k^{\mathrm{T}}(t)(M - d_k^* I_n)\overline{e_k(t)} + \varepsilon\sum_{k=1}^{N}\sum_{l=1}^{N}\left(a_{kl}e_l^{\mathrm{T}}(t)H\overline{e_k(t)} + a_{kl}e_k^{\mathrm{T}}(t)H\overline{e_l(t)}\right)\\
=& E^{\mathrm{T}}(t)\left(I_N \otimes M - D \otimes I_n\right) + \varepsilon(A^{\mathrm{T}} + A) \otimes H\big)\overline{E(t)}
\end{aligned}$$

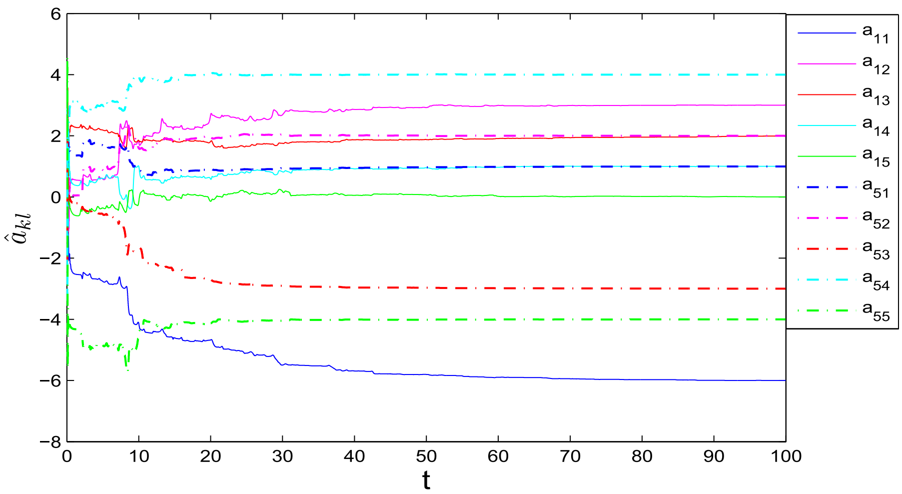

Fig. 1 *Identification of unknown network structure a_{kl}*

The derivative of $V(t)$ along the trajectories of (7) and (8) satisfies

$$\begin{aligned}\dot{V}(t) &= \sum_{k=1}^{N} \dot{e}_k^{\mathrm{T}}(t)\overline{e_k(t)} + \sum_{k=1}^{N} e_k^{\mathrm{T}}(t)\overline{\dot{e}_k(t)} \\ &\quad + \sum_{k=1}^{N} \frac{2(d_k(t) - d_k^*)\dot{d}_k(t)}{\delta_k} \\ &\quad + \sum_{k=1}^{N}\sum_{l=1}^{N} \frac{\varepsilon(\hat{a}_{kl}(t) - a_{kl})\dot{\hat{a}}_{kl}(t)}{\theta_{kl}} \\ &\quad + \sum_{k=1}^{N} \frac{(\hat{\alpha}_k(t) - \alpha_k)^{\mathrm{T}}\dot{\hat{\alpha}}_k(t)}{\eta_k} \\ &\le E^{\mathrm{T}}(t)\big(I_N \otimes M - D \otimes I_n) + (A^{\mathrm{T}} + A) \otimes H\big)\overline{E(t)}\end{aligned}$$

Similar to the proof of the above Theorem 1, one can easily show that $\lim_{t\to\infty} E(t) = 0$ and $\lim_{t\to\infty} \dot{E}(t) = 0$. Then, $y(t) = (y_1^{\mathrm{T}}(t), y_2^{\mathrm{T}}(t), \ldots, y_N^{\mathrm{T}}(t))^{\mathrm{T}}$ converges to the following largest invariant set

$$\mathcal{E} = \left\{ y(t) : F_k(y_k(t))(\hat{\alpha}_k(t) - \alpha_k) + \varepsilon \sum_{l=1}^{N} (\hat{a}_{kl}(t) - a_{kl}) \times Hy_l(t) = 0,\ \ k = 1, 2, \ldots, N \right\}, \quad \text{as } t \to \infty$$

From (A2), one has $\hat{\alpha}_k(t) \to \alpha_k$ and $\hat{a}_{kl}(t) \to a_{kl}$, that is, the unknown system parameters α_k and coupling matrix A are identified when networks (3) and (7) achieve synchronisation. □

Corollary 1: Suppose that (A1) and (A2) hold. If a dynamical network coupled with N identical complex-variable chaotic systems, described by

$$\dot{x}_k(t) = g(x_k(t)) + F(x_k(t))\alpha + \varepsilon \sum_{l=1}^{N} a_{kl}Hx_l(t), \quad k = 1, 2, \ldots, N \tag{9}$$

Then, the unknown system parameters α and coupling matrix A of network (9) can be identified by the the estimated values $\hat{\alpha}$ and $\hat{A}$ of the following network estimator

$$\begin{aligned}\dot{y}_k(t) &= g(y_k(t)) + F(y_k(t))\hat{\alpha}(t) + \varepsilon \sum_{l=1}^{N} \hat{a}_{kl}(t)Hy_l(t) + u_k(t) \\ u_k(t) &= -d_k(t)e_k(t), \quad \dot{d}_k(t) = \delta_k e_k^{\mathrm{T}}(t)\overline{e_k(t)} \\ \dot{\hat{\alpha}}(t) &= -\eta \sum_{k=1}^{N} \left(\overline{F(y_k(t))}^{\mathrm{T}} e_k(t) + F^{\mathrm{T}}(y_k(t))\overline{e_k(t)}\right) \\ \dot{\hat{a}}_{kl}(t) &= -\theta_{kl}(e_k^{\mathrm{T}}(t)H\overline{y_l(t)} + y_l^{\mathrm{T}}(t)H\overline{e_k(t)})\end{aligned}$$

where $k = 1, 2, \ldots, N$, $l = 1, 2, \ldots, N$, $\delta_k > 0$, $\eta > 0$ and $\theta_{kl} > 0$ are constants.

Remark 2: In the above discussions, the coupling matrix A need not to be symmetric and irreducible. The synchronisation and identification speed can be adjusted by choosing proper constants δ_k, η_k and θ_{kl}.

4 Numerical simulations

Consider a dynamical network (3) coupled with five different complex-variable Chen systems [22], which can be described by

$$\dot{x}_k(t) = g_k(x_k(t)) + F_k(x_k(t))\alpha_k + \varepsilon \sum_{l=1}^{5} a_{kl}Hx_l(t), \quad k = 1, 2, \ldots, 5 \tag{10}$$

where $x_k(t) = (x_{k1}(t), x_{k2}(t), x_{k3}(t))^{\mathrm{T}}$ is the state variable, $g_k(x_k) = (0, -x_{k1}x_{k3}, (\overline{x_{k1}}x_{k2} + x_{k1}\overline{x_{k2}})/2)^{\mathrm{T}}$, $\alpha_k = (\alpha_{k1}, \alpha_{k2}, \alpha_{k3})^{\mathrm{T}}$ is the system parameter vector, and

$$F_k(x_k) = \begin{bmatrix} x_{k2} - x_{k1} & 0 & 0 \\ -x_{k1} & x_{k1} + x_{k2} & 0 \\ 0 & 0 & -x_{k3} \end{bmatrix}$$

In the following numerical examples, we choose the coupling matrix A as

$$A = \begin{bmatrix} -6 & 3 & 2 & 1 & 0 \\ 2 & -3 & 3 & -1 & -1 \\ -1 & 3 & -5 & 0 & 3 \\ 0 & 2 & 3 & -4 & -1 \\ 1 & 2 & -3 & 4 & -4 \end{bmatrix}$$

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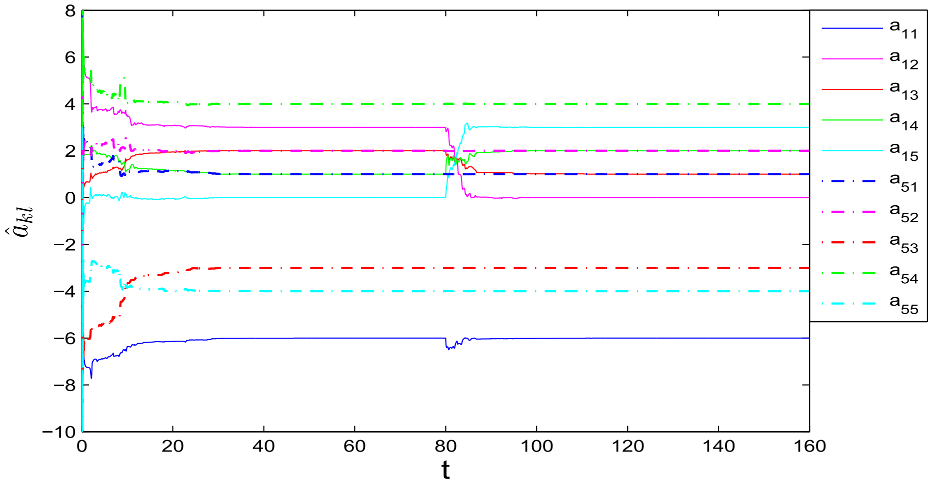

Fig. 2 *Identification of unknown (uncertain) network structure* a_{kl}

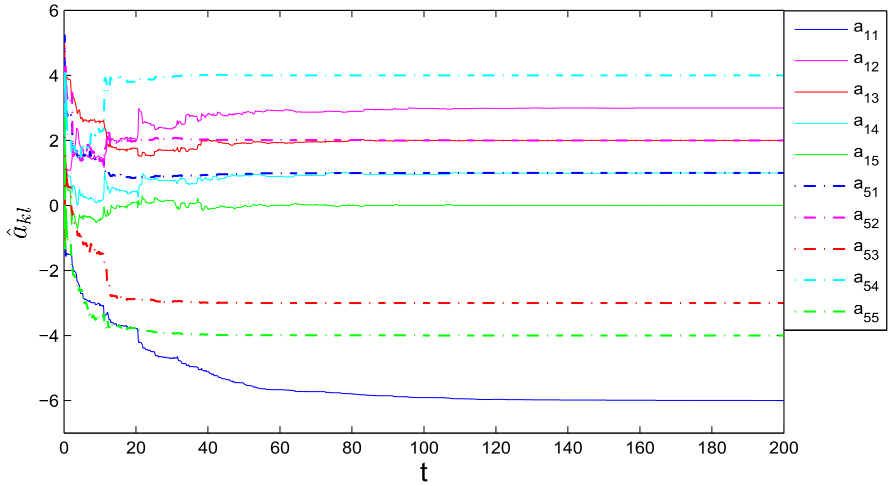

Fig. 3 *Identification of unknown network structure* a_{kl}

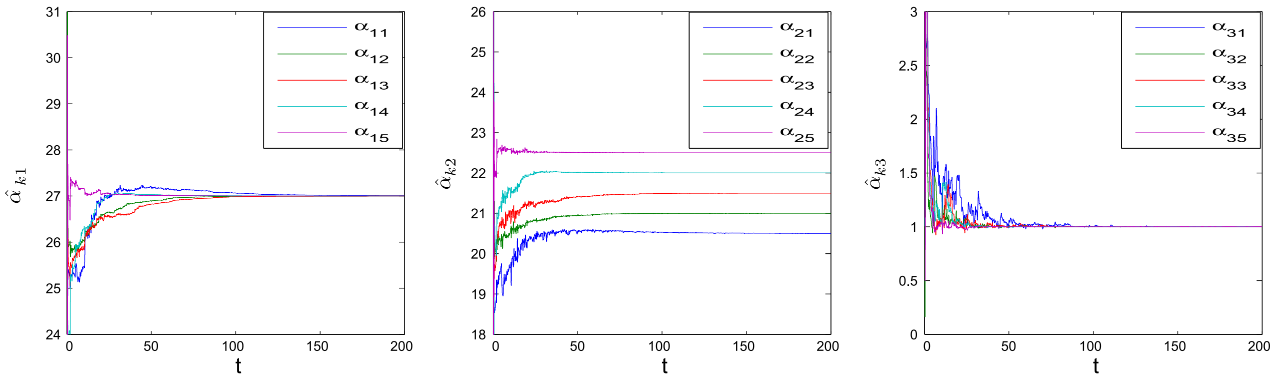

Fig. 4 *Identification of unknown system parameters* α_k

and the system parameter vectors $\alpha_k = (27, 20 + k/2, 1)^{\mathrm{T}}$ for $k = 1, \ldots, 5$.

From [25], the complex-variable Chen systems satisfy Assumption (A1), because they are chaotic and bounded for any initial values. Further, many dynamical networks with non-identical nodes naturally satisfy Assumption (A2) (see [18]), that is, the conditions in above theorems can be satisfied.

First, suppose that only the coupling matrix A is unknown and to be identified. In the numerical simulations, choose the coupling strength $\varepsilon = 0.5$, the inner coupling matrix $H = \mathrm{diag}(1, 1, 1)$, the adaptive gains $\delta_k = 1$ and $\theta_{kl} = 1$, the initial values $d_k(0) = 20$ and $\hat{a}_{kl}(0) = 3$ for $k, l = 1, 2, 3, 4, 5$. Further, choose the initial values of the state variable $x_k(t)$ and $y_k(t)$ randomly. Fig. 1 shows the identification of unknown topology structure.

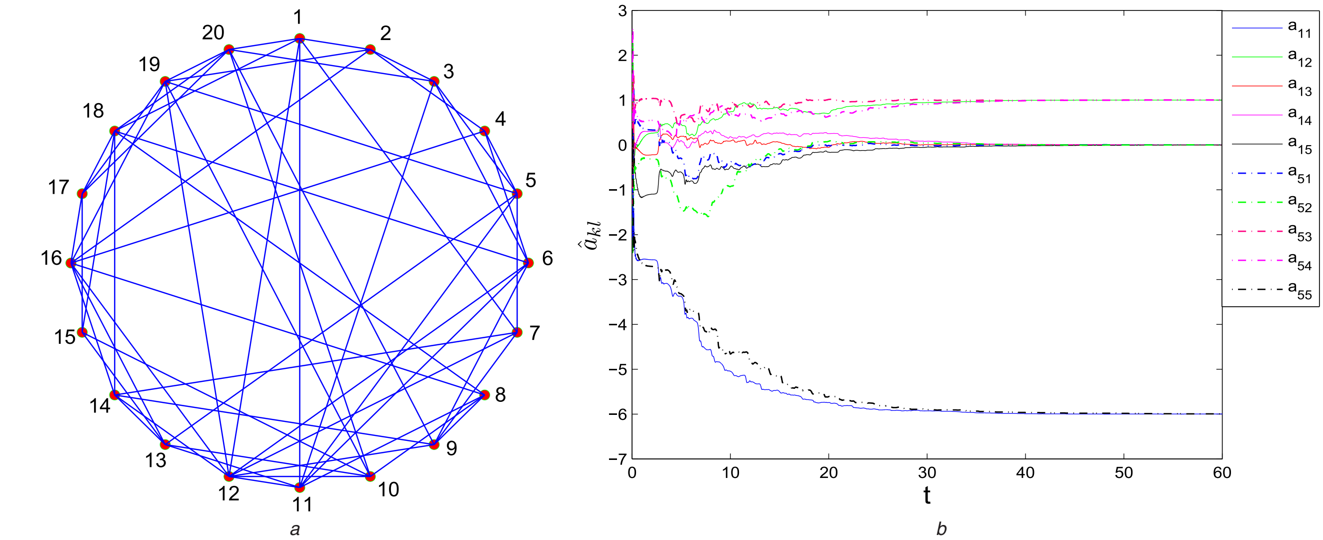

Fig. 5 *Diagrams representing*
a Topological structure of network (9)
b Identification of unknown network structure a_{kl}

Second, suppose that the coupling matrix A is uncertain and switched from A to the following matrix A' at time $t = 80$

$$A' = \begin{bmatrix} -6 & 0 & 1 & 2 & 3 \\ 2 & -3 & 3 & -1 & -1 \\ -1 & 3 & -5 & 0 & 3 \\ 0 & 2 & 3 & -4 & -1 \\ 4 & -3 & 2 & 1 & -4 \end{bmatrix}$$

In numerical simulations, choose the same parameters as those in the above example. Fig. 2 shows that the network estimator can not only identify the unknown network structure but also monitor the switching structure rapidly.

Third, suppose that both the coupling matrix A and the system parameters α_k are unknown and to be identified. In numerical simulations, choose the same parameters as those in the above example. Further, choose the initial values of estimated values $\hat{\alpha}_k(t)$ as $\alpha_{k1}(0) = 25$, $\alpha_{k2}(0) = 22$, $\alpha_{k3}(0) = 2$ and $\eta_k = 3$, $k = 1, 2, 3, 4, 5$. Fig. 3 shows the identification of unknown topology structure and Fig. 4 shows the identification of unknown system parameters.

Finally, consider network (9) consisting of 20 identical complex-variable Chen systems with WS small-world topological structure, which is shown in Fig. 5*a*. Suppose that only the coupling matrix A is unknown and to be identified. In numerical simulations, choose the coupling strength $\varepsilon = 0.1$, the inner coupling matrix $H = \mathrm{diag}(1, 1, 1)$, the adaptive gains $\delta_k = 1$ and $\theta_{kl} = 1$, the initial values $d_k(0) = 20$ and $\hat{a}_{kl}(0) = 0.5$ for $k, l = 1, 2, \ldots, 20$. Then choose the initial values of the state variable $x_k(t)$ and $y_k(t)$ randomly. Fig. 5*b* shows the identification of unknown topological structure.

5 Conclusions

This paper discusses structure identification of uncertain dynamical networks coupled with complex-variable chaotic systems. Based on the Barbalat's lemma, an adaptive control scheme is used to design corresponding network estimators for identifying the unknown or uncertain topological structures and node dynamics. In particular, the topological structure need not to be symmetric or irreducible, and the local node dynamics need not to be identical. Furthermore, numerical simulations show that this method can monitor the switching topological structures of a dynamical network quickly.

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7 References

1 Newman, M.E.J.: 'The structure and function of complex networks', *SIAM Rev.*, 2003, **45**, pp. 167–256
2 Boccaletti, S., Latora, V., Moreno, Y., Chavez, M., Hwang, D.U.: 'Complex networks: Structure and dynamics', *Phys. Rep.*, 2006, **424**, pp. 175–308
3 Arenas, A., Díaz-Guilera, A., Kurths, J., Moreno, Y., Zhou, C.: 'Synchronization in complex networks', *Phys. Rep.*, 2008, **469**, pp. 93–153
4 Yu, W., Cao, J., Lü, J.: 'Global synchronization of linearly hybrid coupled networks with time-varying delay', *SIAM, J. Appl. Dyn. Syst.*, 2008, **7**, pp. 108–133
5 Jiang, H.B.: 'Hybrid adaptive and impulsive synchronisation of uncertain complex dynamical networks by the generalised Barbalat's lemma', *IET Control Theory Appl.*, 2209, **3**, pp. 1330–1340
6 Wang, Z., Liao, X., Guo, S., Liu, G.: 'Stability analysis of genetic regulatory network with time delays and parameter uncertainties', *IET Control Theory Appl.*, 2010, **4**, pp. 2018–2028
7 Yang, Z., Zhang, Q., Chen, Z.: 'Adaptive linear generalized synchronization between two nonidentical networks', *Commun. Nonlinear Sci. Numer. Simul.*, 2012, **17**, pp. 2628–2636
8 Dai, Y., Cai, Y., Xu, X.: 'Synchronization analysis and impulsive control of complex networks with coupling delays', *IET Control Theory Appl.*, 2009, **3**, pp. 1167–1174
9 Gómez-Gardeñes, J., Gómez, S., Arenas, A., Moreno, Y.: 'Explosive synchronization transitions in scale-free networks', *Phys. Rev. Lett.*, 2011, **106**, article id 128701

10 Pan, L., Cao, J.: 'Stochastic quasi-synchronization for delayed dynamical networks via intermittent control', *Commun. Nonlinear Sci. Numer. Simul.*, 2012, **17**, pp. 1332–1343
11 Wang, X., Chen, G.: 'Pinning control of scale-free dynamical networks', *Physica A*, 2002, **310**, pp. 521–531
12 Yu, W., Chen, G., Lü J.: 'On pinning synchronization of complex dynamical networks', *Automatica*, 2009, **45**, pp. 429–435
13 Song, Q., Cao, J.: 'On Pinning synchronization of directed and undirected complex dynamical networks', *IEEE Trans. Circuits Syst. I*, 2010, **57**, pp. 672–680
14 Yu, D., Righero, M., Kocarev, L.: 'Estimating topology of networks', *Phys. Rev. Lett.*, 2006, **97**, article id 188701
15 Yu, W., Chen, G., Cao, J., Lü J., Parlitz, U.: 'Parameter identification of dynamical systems from time series', *Phys. Rev. E*, 2007, **75**, article id 067201
16 Wu, X.: 'Synchronization-based topology identification of weighted general complex dynamical networks with time-varying coupling delay', *Physica A*, 2008, **387**, pp. 997–1008
17 Gan, Q.: 'Synchronisation of chaotic neural networks with unknown parameters and random time-varying delays based on adaptive sampled-data control and parameter identification', *IET Control Theory Appl.*, 2012, **6**, pp. 1508–1515
18 Liu, H., Lu, J., Lü J., Hill, D.J.: 'Structure identification of uncertain general complex dynamical networks with time delay', *Automatica*, 2009, **45**, pp. 1799–1807
19 Zhao, J., Li, Q., Lu, J., Jiang, Z.: 'Topology identification of complex dynamical networks', *Chaos*, 2010, **20**, article id 023119
20 Xu, Y., Zhou, W., Fang, J., Sun, W.: 'Topology identification and adaptive synchronization of uncertain complex networks with adaptive double scaling functions', *Commun. Nonlinear Sci. Numer. Simul.*, 2011, **16**, pp. 3337–3343
21 Fowler, A.C., Gibbon, J.D., McGuinness, M.J.: 'The complex Lorenz system', *Physica D*, 1982, **4**, pp. 139–163
22 Mahmoud, G.M., Bountis, T., Mahmoud, E.E.: 'Active control and global synchronization for complex Chen and Lü systems', *Int. J. Bifurcation Chaos*, 2007, **17**, pp. 4295–4308
23 Mahmoud, G.M., Aly, S.A., AL-Kashif, M.A.: 'Dynamical properties and chaos synchronization of a new chaotic complex nonlinear system', *Nonlinear Dyn.*, 2008, **51**, pp. 171–181
24 Mahmoud, G.M., Aly, S.A., Farghaly, A.A.: 'On chaos synchronization of a complex two coupled dynamos system', *Chaos Solitons Fractals*, 2007, **33**, pp. 178–187
25 Wu, Z., Chen, G., Fu, X.: 'Synchronization of a network coupled with complex-variable chaotic systems', *Chaos*, 2012, **22**, 023127
26 Khalil HK: 'Nonlinear systems' (Prentice-Hall, Englewood Cliffs, NJ, 2002)