Summer Seminar in Mathematics (夏季数学讲座)
Xiayuan 310 (下院310)
Tue, Thu, 14:00–15:40, 16:00–17:40

> Dare to know! Have the courage to use your own intelligence! -- Immanuel Kant, Answering the Question: What Is Enlightenment? 1784.
> http://www.columbia.edu/acis/ets/CCREAD/etscc/kant.html
> http://gutenberg.spiegel.de/buch/-3505/1
> https://en.wikipedia.org/wiki/Sapere_aude

> Good mathematics has an air of economy and an element of surprise. -- Ian Stewart, From Here to Infinity: A Guide to Today's Mathematics, Oxford University Press, Revised edition, 1996.
> https://en.wikipedia.org/wiki/From_Here_to_Infinity_(book)

> Each day offers a different theorem (or lemma, law, formula or identity), each one worthy of adorning the walls of a mathematical Abattoirs, Baltic, Cairn House, Guggenheim, Louvre, Nail Factory, Staatliche Museen, Tate, Uffizi or Zach Feuer.
> Each theorem has been presented so as to be appreciated by as wide an audience as possible. If the statement of the theorem appears obscure, pass on to the illustration and its accompanying explanation. If it still seems hard it is probably because it is hard. But no more than O'Keeffe's Blue and Green Music, say, is a 'hard' painting or Hepworth's Two Figures is a 'hard' sculpture. It is there for you to engage with on your own terms.
> By 'engage' I mean: admire it, turn it over in your mind, try to follow the example, if one is given; if you are studying it on-line, follow the web link, which will provide a pictorial interpretation, a proof or even a clever animation. -- Robin Whitty, http://www.theoremoftheday.org/

> The measure of our success is whether what we do enable people to understand and think more clearly about mathematics. -- William P. Thurston, On proof and progress in mathematics, Bull. Amer. Math. Soc. 30 (1994), 161-177. http://www.ams.org/journals/bull/1994-30-02/

June 28, 14:00–15:40, Yaokun Wu (吴耀琨) ✍ **ykwu@sjtu.edu.cn**

三戏信息流

Three games on information flows

This talk introduces three games which can be played by elementary school students. If you have become a good player on any of them, you may realize some surprising connections among the three and you may be counted as a good expert in coding theory, dynamical system, circuit complexity, probabilistic models, and some other mathematics fields. Most part of this expository talk requires no mathematical knowledge. When time allows, we may indicate a bit some nice applications of linear algebra in analysing these games.

- R. Ahlswede, N. Cai, S-Y. Li, R.W. Yeung, Network information flow, IEEE Trans. Inform. Theory 46 (2000) 1204–1216.
- F. Arbabjolfaei, Y-H. Kim, Three stories on a two-sided coin: Index coding, locally recoverable distributed storage, and guessing games on graphs, Proceedings of the 53rd Annual Allerton Conference on Communication, Control, and Computing, Monticello, Illinois, October 2015.
- R. Atkins, The information flow problem on clock networks, http://arxiv.org/abs/1605.05391 .
- Z. Bar-Yossef, Y. Birk, T.S. Jayram, T. Kol, Index coding with side information, IEEE Trans. Inform. Theory 57 (2011) 1479–1494.
- M. Gadouleau, Maximum rank and asymptotic rank of finite dynamical systems, http://arxiv.org/abs/1512.01448v1 .
- M. Gadouleau, A. Richard, S. Riis, Fixed points of Boolean networks, guessing graphs, and coding theory, SIAM Journal on Discrete Mathematics 29 (2015) 2312–2335.
- S. Riis, Information flows, graphs and their guessing numbers, Electronic Journal of Combinatorics, 14:R44, 2007.

June 28, 16:00–17:40, Mijia Lai (来米加) ✍ **laimijia@sjtu.edu.cn**

从长度、面积、体积到测度

From length, area and volume to measure

As illustrated in the title, in this lecture we will give a brief introduction to the measure theory. Roughly speaking, measure generalizes the most familiar notions of length, area and volume in Euclidean geometry to arbitrary subsets in $\mathbb{R}^n$. When doing this, some bizarre-looking things occur, but they are mathematically fascinating. If time permits, we shall talk about a powerful tool: the Steiner symmetrization.

June 30, 14:00 – 15:40, Yuehui Zhang (张跃辉) ✍ **zyh@sjtu.edu.cn**

The Riemann Hypothesis

素数的音乐—黎曼假设简介

被誉为“天籁之音”的黎曼假设源自1859年柏林科学院新科院士黎曼的题为“论不超过给定实数之素数个数”的研究报告. 黎曼假设是指黎曼在该文中的著名推断, 即现被称为“黎曼ζ- 函数”的复变函数

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}$$

的所有零点除全体负偶数外均在临界线 $Re(s) = \frac{1}{2}$ 上.
“黎曼ζ- 函数”源自调和级数, 而调和级数的源头正是音乐中的和弦. 这一点并非偶然, 因为数学乃音乐之魂. 黎曼天才地认识到理解数学之原子—素数的关键在于弃实数之暗投复数之明, 于是 $\sum_{n=1}^{\infty} \frac{1}{n^s}$ 对所有 $s \neq 1$ 均有意义, 因此ζ-函数除了“1”这唯一的“坏点”之外, 是个非常好的函数, 特别地, 黎曼证明了ζ-函数满足对称性方程, 进而可知ζ-函数具有无限多个零点, 包括所有的负偶数! 于是黎曼的复数世界缤纷绚烂乃至惊世骇俗:

$$1^2 + 2^2 + 3^2 + ... + n^2 + ... = 0! \quad 1^{2k} + 2^{2k} + 3^{2k} + ... + n^{2k} + ... = 0!$$

(其中 k 为正整数.) 黎曼将此种现象称之为“平凡”, 因为他拥有一双“复眼”–复数的眼睛！ 经过150多年的人机协力, 得益于计算机之父图灵的奇妙算法, 今天我们知道至少40%的零点以及最前面 $10^{22} + 10^4$ (此数大于“一百万亿亿”)个零点均在临界线上, 但所有这些艰苦努力似乎距离终点遥遥无期, 因为数学家的研究方法尚未突破黎曼一个半世纪以前的思想, 因为人类对与实数序结构(即大小关系)大相径庭的复数序结构的理解仍停留在黎曼时代. 然而物理世界早已天翻地覆, 物理学家于是跨界来帮助几乎走投无路的数学界同仁, 他们的武器是上世纪八十年代发现的“准晶”, 他们的信念是黎曼ζ- 函数的非平凡零点构成一个“1- 维准晶”(现实世界中当然没有此种东东), 因此1- 维准晶理论的完成之日即黎曼假设的证明之时.

June 30, 16:00 – 17:40, Eiichi Bannai (坂内英一) ✍ **bannai@sjtu.edu.cn**

亲吻数和最密球堆积

Kissing number and densest sphere packing

We will discuss the following two famous problems.

(i) The kissing number problem. For a given unit sphere in the real Euclidean space $\mathbb{R}^n$, how many non-overlapping unit spheres can be put to touch the given unit sphere? The maximum number of such outside spheres is called the kissing number $k(n)$ for $\mathbb{R}^n$. It is obvious that $k(2) = 6$. We study what are $k(n)$ for $n \geq 3$.

(ii) The sphere packing problem. We want to fill $\mathbb{R}^n$ with (infinitely many) unit spheres. What is the best density $\Delta(n)$ of the part covered by the spheres? It is known that $\Delta(2) = \pi/\sqrt{12}$. We study what are $\Delta(n)$ for $n \geq 3$.

July 5, 14:00 – 15:40, Weike Wang (王维克) ✍ **wkwang@sjtu.edu.cn**

Functions of now and before

函数的前世今生

函数是数学中最寻常的一个概念，但从历史上看，函数概念的深化和改变往往伴随着数学观念的革命性进步。本讲从经典的函数定义出发到从“基”或者“对偶”的观点看函数，希望从函数的“前世今生”一窥现代分析学一些不同侧面的概貌。

July 5, 16:00 – 17:40, Feng Xie (谢峰) ✍ **tzxief@sjtu.edu.cn**

可压缩流体中的基本波现象: I

Elementary wave phenomena in compressible flows: I

In this lecture, we will give several examples about the elementary wave (Shock wave, rarefaction wave etc.) phenomena in real life. How to understand and study these important waves from the mathematical point of view? It is an interesting and serious thing we need to think about.

July 7, 14:00 – 15:40, Yi Li (李逸) ✍ **yilicms@sjtu.edu.cn**

测度度量空间简介

Introduction to metric measure spaces

In this topic, I will introduce the concept of metric measure space, which is a natural generalization of the Euclidean space. We also discuss the basic properties of metric measure space and the applications to geometry.

July 7, 16:00 – 17:40, Feng Xie (谢峰) ✍ **tzxief@sjtu.edu.cn**

可压缩流体中的基本波现象: II

Elementary wave phenomena in compressible flows: II

In this lecture, we will give several examples about the elementary wave (Shock wave, rarefaction wave etc.) phenomena in real life. How to understand and study these important waves from the mathematical point of view? It is an interesting and serious thing we need to think about.

July 12, 14:00 – 15:40, Yaokun Wu (吴耀琨) ✍ **ykwu@sjtu.edu.cn**

什么是杨表?

What is a Young tableau?

Young tableaux are ubiquitous combinatorial objects making important and inspiring appearances in representation theory, geometry, and algebra. This talk aims to introduce some interesting facts about Young tableaux as well as their elementary proofs.

- V.I. Danilov, G.A. Koshevoy, Arrays and the combinatorics of Young tableaux, Russian Mathematical Surveys, 60 (2005), 79–142
- William Fulton. Young Tableaux, with Applications to Representation Theory and Geometry. Cambridge University Press, 1997.
- F. Hivert, J.-C. Novelli, J.-Y. Thibon, The algebra of binary search trees, Theoretical Computer Science 339 (2005) 129–165.
- R. Steinberg, An occurrence of Robinson-Schensted correspondence, Journal of Algebra 113 (1988) 523–528.
- Alexander Yong, What is a Young tableau? Notices of the AMS 54 (2007) 240–241. http://www.ams.org/notices/200702/whatis-yong.pdf

July 12, 16:00–17:40, Yongzhong Xu (徐永忠) ✍ **abbyxu@gmail.com**

莫尔斯理论简介

An introduction to Morse theory

Morse theory is a beautiful theory which connects the topology of a manifold with the property of critical points of functions defined on it. I will describe the story following Milnor's proof. Then I will briefly describe another approach initiated by Witten and Floer.

- R. Bott, Morse Theory indomitable, Publications Mathématiques de l'IHÉS, 68 (1988) 99–114.
- J. Milnor, Morse Theory, Princeton University Press, 1963.
- 米尔诺(John W. Milnor) (著)，熊金城(译)，从微分观点看拓扑(双语版)，人民邮电出版社，2008.

July 14, 14:00 – 15:40, Lei Zhang (张镭) ✍ **lzhang2012@sjtu.edu.cn**

地球物理与材料科学中的数学问题: I

Mathematics problems in geophysics and materials science: I

I am going to introduce the mathematical modelling, analysis and simulation for applications in geophysics and materials science. For example, seismic imaging and reservoir modelling in geophysics, as well as multi-scale simulation methods and materials genome project in materials science.

July 14, 16:00 – 17:40, Yi Li (李逸) ✍ **yilicms@sjtu.edu.cn**

几何群论简介

Introduction to geometric group theory

In this topic we shall discuss the geometric groups and try to give a self-contained proof of the famous Gromov theorem.

I only assume the students have a good background on calculus and linear algebra.

July 19, 14:00 – 15:40, Mijia Lai (来米加) ✍ **laimijia@sjtu.edu.cn**

射影空间和裴蜀定理

Projective space and Bezout's theorem

In this lecture, we shall give a short introduction to algebraic geometry, which in simple terms, is the study of zeros of polynomials. After introducing the projective spaces, we shall prove two classical theorems on intersection theory: Bezout's theorem and Cayley-Bacharach theorem. We will see how they yield a high-tech proof of Pappus and Pascal's theorems in classical Euclidean geometry.

July 19, 16:00 – 17:40, Lei Zhang (张镭) ✍ **lzhang2012@sjtu.edu.cn**

地球物理与材料科学中的数学问题: II

Mathematics problems in geophysics and materials science: II

I am going to introduce the mathematical modelling, analysis and simulation for applications in geophysics and materials science. For example, seismic imaging and reservoir modelling in geophysics, as well as multi-scale simulation methods and materials genome project in materials science.

July 21, 14:00 – 15:40, Jiyou Li (李吉有) ✍ **lijiyou@sjtu.edu.cn**

音乐与数学

Music and mathematics

Pythagoras said, "There is geometry in the humming of the strings, and there is music in the spacing of the spheres." That means music can be understood geometrically. As the most abstract art form, music has many similarities to mathematics. For instance, pitches are represented by numbers, chords are represented by sets and the harmony of an interval is related to a rational number.

In this lecture we will discuss these connections, including the basic theory of scales and the geometry of chords.

July 21, 16:00 – 17:40, Yongzhong Xu (徐永忠) ✍ **abbyxu@gmail.com**

一对姐妹花：偶数维世界的辛几何与奇数维世界的切触几何

Symplectic geometry and its odd-dimensional twin contact geometry

I will first describe what is a symplectic form and how it is related to Newton mechanics etc. then probably the Darboux lemma which tells us every symplectic manifold looks alike locally, then the rigidity result which tells us symplectic manifold indeed is different with the usual thing. Then describe what is a contact form and the Weinstein conjecture. I will design the talk to be accessible to freshmen.

- Dusa McDuff, http://www.math.stonybrook.edu/ dusa/princerev98.pdf
- 阿诺尔德(V.I. Arnold) (著)，齐民友 (译)，经典力学中的数学方法，高等教育出版社，2006.

Some books in the spirit of this seminar series, plus random pieces of their reviews from http://www.ams.org/mathscinet/

Gilbert Strang, Essays in Linear Algebra, Wellesley-Cambridge Press, 2012.

This book brings together 27 classic articles by Gilbert Strang on linear algebra, computational science, applied mathematics, and calculus, each introduced by a new essay. The essays present historical background, the current state, and unsolved problems.

Aigner, Martin (D-FUB-MI) Markov's theorem and 100 years of the uniqueness conjecture. A mathematical journey from irrational numbers to perfect matchings. Springer, Cham, 2013. x+257 pp. ISBN: 978-3-319-00887-5; 978-3-319-00888-2

Toth, Gabor (1-RTG) Glimpses of algebra and geometry. (English summary) Second edition. Undergraduate Texts in Mathematics. Readings in Mathematics. Springer-Verlag, New York, 2002. xxii+450 pp. ISBN: 0-387-95345-0

Serge Lang, Challenges, Springer; Softcover reprint of the original 1st ed. 1998 edition (October 4, 2013).

Serge Lang, Math Talks for Undergraduates, Springer, 1999.

Anyhow, thinking about mathematics is very pleasurable to many people, and there is considerable evidence that we are programmed naturally to like mathematics, until the pleasure is ruined by incompetent teaching or other social factors. Every 5 year old kid I have ever met likes to add numbers, or subtract numbers. What happens afterwards is something else. – S. Lang, Foreword to the book.

Laczkovich, Miklós (H-EOTVO) Conjecture and proof. (English summary) Classroom Resource Materials Series. Mathematical Association of America, Washington, DC, 2001. x+118 pp. ISBN: 0-88385-722-7

According to the author, Miklós Laczkovich (of circle-squaring fame and recipient of the Ostrowski Prize in 1993 for his work in 1989 on the equidecomposability of a circle with a square of the same area), this book is "an elaborate version of the lecture notes for a one-semester course of the Budapest Semesters in Mathematics (BSM)" with the same course title as that of the book. The BSM program, which was initiated and designed by Paul Erdös, László Lovász, Vera T. Sós and László Babai in 1983 – 1984, offers to American and Canadian visiting students undergraduate courses that convey the Hungarian tradition in mathematics. In particular, this course titled "Conjectures and Proofs" is based on creative problem solving and thereby expounds a number of interesting and important topics to quite some depth but from first principles.

Aigner, Martin(D-FUB-NDM); Ziegler, Günter M.(D-FUB-NDM) Proofs from The Book. Fifth edition. Including illustrations by Karl H. Hofmann. Springer-Verlag, Berlin, 2014. viii+308 pp. ISBN: 978-3-662-44204-3; 978-3-662-44205-0

Stillwell, John(1-SNFR) Mathematics and its history. Third edition. Undergraduate Texts in Mathematics. Springer, New York, 2010. xxii+660 pp. ISBN: 978-1-4419-6052-8

Bollobás, Béla(4-CAMBT) The art of mathematics. Coffee time in Memphis. Cambridge University Press, New York, 2006. xvi+359 pp. ISBN: 978-0-521-69395-0; 0-521-69395-0

Wiener, Norbert I am a mathematician. The later life of a prodigy. Doubleday and Co., Garden City, N. Y., 1956. 380 pp.

Adams, Colin C. The knot book. An elementary introduction to the mathematical theory of knots. Revised reprint of the 1994 original. American Mathematical Society, Providence, RI, 2004. xiv+307 pp. ISBN: 0-8218-3678-1

Conway, John H.(1-PRIN) The sensual (quadratic) form. With the assistance of Francis Y. C. Fung. Carus Mathematical Monographs, 26. Mathematical Association of America, Washington, DC, 1997. xiv+152 pp. ISBN: 0-88385-030-3

Ronan, Mark(1-ILCC) Symmetry and the monster. One of the greatest quests of mathematics. Oxford University Press, Oxford, 2006. vi+251 pp. ISBN: 978-0-19-280722-9; 0-19-280722-6

This book tells the story of finite simple groups over nearly two and a half millenia, from the almost prehistoric classification of the Platonic solids to the complete classification of the finite simple groups. It is an ambitious attempt to explain the fascination of the subject to the lay reader while at the same time introducing the basic mathematical ideas with a minimum of jargon. There is a wealth of historical anecdotes and entertaining stories to sugar the mathematical pill, and even the reader unable to swallow the latter will have enjoyed the former. The story really begins, of course, with Galois. After a discussion of what symmetry is, and a brief history of solutions of quadratic, cubic and quartic equations, comes a description of Galois theory at the level of the 'group of permutations of the roots of the equation'. The author makes a serious effort to explain what groups are, and what simple groups are, including a construction of the alternating groups.

Hilton, Peter(1-SUNY2); Holton, Derek(NZ-OTG); Pedersen, Jean(1-STCL) Mathematical reflections. (English summary) In a room with many mirrors. Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1997. xvi+351 pp. ISBN: 0-387-94770-1

The aim of this book is to present an authentic view of mathematics and, at the same time, draw the reader into an active role. The first eight chapters, which can be read independently, treat a variety of topics: spirals; number theory; the Fibonacci sequence; polygons through paper folding; quilts, symmetry and tesselations; Pascal's triangle; infinity; dynamics and fractal geometry. In the ninth and final chapter, the authors provide advice for would-be mathematicians.

William J. Cook, In Pursuit of the Traveling Salesman: Mathematics at the Limits of Computation, Princeton University Press; First Edition edition (January 16, 2012).

"A gripping insider's account of one of the great mathematical problems. This book shows how deep mathematical insights can arise from apparently simple questions, and how the results can be applied to that most human of objectives: to achieve a desired outcome in the best possible way. In Pursuit of the Traveling Salesman deserves to become an instant classic." – Ian Stewart,

Halmos, Paul R.(1-STCL) I want to be a mathematician. An automathography. Springer-Verlag, New York, 1985. xvi+421 pp. ISBN: 0-387-96078-3

This autobiography is a frank, personal, witty commentary on mathematicians and mathematics by one of the most influential, and observant, mathematicians of our time. It is much more about the profession of mathematics than about the personal life of its author.

We are surprised at how old-fashioned the college and even graduate school curriculum was in the thirties. We realize that he helped enormously to change that curriculum. He tells how difficult the steps were for him that led to being a mathematician. We see the famous books emerge from his enthusiasm for each new field he enters. All along the way we are treated to anecdotes and photographs of the great and the less great as Halmos met them. He does not say what it feels like now to be a father figure to younger generations.

He makes two main points. The first is the importance of being literate. The ability to speak and write effectively, preferably in more than one language, is essential to effectiveness in all professional activity. And second, a real professional must work in all aspects of the job: research of course, but also teaching in several formats, exposition at several levels, refereeing and editing, departmental chores, and participation in meetings and conferences. His standard is high. Halmos' many friends agree that he himself is a very good mathematician indeed in each of these domains. But in describing the complete professional he seems to denigrate the value of partial talent. Some researchers cannot teach, others are bad referees, many are intimidated or frustrated at meetings and conferences. We admire those who are universally gifted and energetic, but it takes many more to do the work of our calling.
Reviewed by Henry Helson

Stephen M. Stigler, The Seven Pillars of Statistical Wisdom, Harvard University Press (March 7, 2016).

Siobhan Roberts, King of Infinite Space: Donald Coxeter, the Man Who Saved Geometry, Walker Books; First Edition edition (September 5, 2006).

Lance Fortnow, The Golden Ticket: P, NP, and the Search for the Impossible, Princeton University Press; 1st edition (March 31, 2013)

Frenkel, Edward (1-CA) Love and Math. The heart of hidden reality. Basic Books, New York, 2013. v+292 pp. ISBN: 978-0-465-05074-1; 978-0-465-06995-8

Years ago, attending one of my first national mathematics meetings in Louisville and having brought along my wife to window shop there, I persuaded her to accompany me to hear a professor from whose texts I had studied countless hours. The auditorium was packed with thousands, and my excitement rose as the speaker was introduced. But after a minute into what I had long anticipated, what with the speaker's monotonic voice, illegibly scrawled overheads, and lackluster body language—he never once looked away from his materials at the podium—I turned to my bride and promised, "I will never again invite you to a math talk".

In his book Love and math, Frenkel suggests "that perhaps we should jail some of the leading mathematicians to force them to express their ideas in accessible terms". His book is an exercise in trying to make the world of math and mathematicians somewhat accessible to those "without any background in mathematics", "to put things in the simplest and clearest possible way", and to show the reader "the connections between [mathematical] objects and the creative process [of mathematicians]: what drives them and how they learn from each other". In a word, Frenkel wants to unveil the passion or love of doing mathematics.

Does he succeed?

In his narrative Frenkel has meshed together three distinct themes: a heartwarming autobiography, a low-key introduction to modern mathematics, and a "mind-boggling" quest for a unified theory of mathematics. Frenkel scores well in describing his youth: of being mentored by a local mathematician while yet in a secondary school two hours by train from Moscow, of encountering anti-Semitism in being denied a fair chance at matriculation into the leading Moscow university, of being nevertheless "turbo-charged" by gifted, bold, and kind-hearted mathematicians throughout his undergraduate days at Moscow's Institute of Oil and Gas, which in turn led to notable math publications, which in the early days of perestroika led to a fellowship opportunity at Harvard while he was yet 21 years of age, and so on to being a world class mathematician.
...

"It's hard work being a teacher [of mathematics]. It's like having children. You sacrifice a lot, not asking for anything in return. How do you decide in which direction to point students, when to give them a helping hand? This is an art. No one can teach you how to do this." Reviewed by Andrew James Simoson

Michael Harris, Mathematics without Apologies: Portrait of a Problematic Vocation, Princeton University Press, 2015.

Siobhan Roberts, Genius At Play: The Curious Mind of John Horton Conway, Bloomsbury USA (July 14, 2015).

Stewart, Ian(4-WARW) From here to infinity. (English summary) A retitled and revised edition of The problems of mathematics [Oxford Univ. Press, New York, 1992; MR1193918]. With a foreword by James Joseph Sylvester. The Clarendon Press, Oxford University Press, New York, 1996. xii+310 pp. ISBN: 0-19-283202-6

From the preface: "Mathematics is changing very rapidly indeed. You can tell that just by following the career of this book, which first saw daylight in 1987 under the title The problems of mathematics. By 1992 several fundamental new discoveries had made a second edition imperative. The title remained the same, but the cover became more jazzy, with a fancy computer graphic in place of a design made from coloured pins. And now it has become necessary to produce a third edition, with a user-friendly title and a distinct shift of emphasis. The new title, by the way, is supposed to indicate that mathematics combines relevance to everyday life ('here') with sweeping intellectual invention ('infinity'). The new emphasis is more on the overall ebb and flow of mathematics, and less on particular problems.

Timothy Gowers (Ed.), The Princeton Companion to Mathematics, Princeton University Press, 2008.

Mark Kac, Gian-Carlo Rota, Jacob T. Schwartz, Discrete Thoughts: Essays on Mathematics, Science and Philosophy, Birkhäuser; 2nd edition (January 11, 2008).

Gian-Carlo Rota, Indiscrete Thoughts, Birkhäuser; 3rd printing 2000 edition (December 18, 1996).

V. I. Arnold, Mathematical Understanding of Nature: Essays on Amazing Physical Phenomena and Their Understanding by Mathematicians, American Mathematical Society (September 4, 2014).

Steven G. Krantz, The Survival of a Mathematician, American Mathematical Society (December 22, 2008).

Franco Vivaldi, Mathematical Writing, Springer Undergraduate Mathematics Series, Springer; 2014 edition (December 14, 2014).

Shurman, Jerry (1-REED) Geometry of the quintic. A Wiley-Interscience Publication. John Wiley & Sons, Inc., New York, 1997. xii+200 pp. ISBN: 0-471-13017-6

Boris A. Khesin, Serge L. Tabachnikov, Arnold: Swimming Against the Tide, American Mathematical Society (September 10, 2014).

Meaning in mathematics. Edited by John Polkinghorne. Oxford University Press, Oxford, 2011. xii+159 pp. ISBN: 978-0-19-960505-7 00A30 (00A35 00B15)

What, if anything, do mathematical propositions mean? This is a rich topic in the philosophy of mathematics. Whole books have been written about it. Meaning in mathematics is not one of them. In spite of the title, the question of what mathematical propositions mean receives scant attention in the volume. The clear exceptions are few: Stewart Shapiro remarks that disagreements about axioms can be interpreted as disagreements about what those axioms mean (p. 104) and Gideon Rosen briefly considers whether mathematical languages are semantically indeterminate, that is, whether a mathematical theorem can advance more than one true claim (p. 130). More in the spirit of the volume, Rosen earlier mentions linguistic meaning only to warn us that he is not then discussing it (p. 122). The editor himself says that the book addresses the following two questions. Is mathematical activity a matter of discovery or invention? How can we best understand the utility of mathematics in empirical science? "How one answers these questions", he says, "will significantly influence metaphysical thinking about reality" (p. 1). The editor suggests that his own answers support a belief in supernatural beings. So perhaps, after all, the book was meant to be about meaning: not linguistic meaning, but meaning as in "the meaning of life". The book is the fruit of a symposium sponsored by the John Templeton Foundation, an organization that styles itself "a philanthropic catalyst for discoveries relating to the Big Questions of human purpose and ultimate reality". Perhaps the editor hoped that, by exposing mathematics as a science of the supernatural, the symposiasts would encourage an expansive conception of human purpose. A better title for the book might then have been Meaningfulness through mathematics. Really though, the editor, an intelligent man with a rich experience of academic life, must have expected the symposiasts to go off every which way—as, in fact, they did.

The literary quality of the book is generally high—even if it does stumble out the blocks with the pleonasm "unanimous wish of all of us" on the very first page. There are occasional bloopers. (The kitchen that figures in an analogy on page 37 becomes a table on page 39. The imaginary Pat, who endorses excluded middle at the top of page 105, is an intuitionist just a few lines later.) On the whole, though, the book is well produced. As for substance, we get just what we would expect from a collection of symposium papers: some coasting, some self-promotion, and some intelligent discussion of important issues. This reviewer benefited most from the contributions of Rosen, Shapiro, and Michael Detlefsen.

Kenji Ueno, Koji Shiga, Shigeyuki Morita, Toshikazu Sunada, A Mathematical Gift, I, II, III: The Interplay Between Topology, Functions, Geometry, and Algebra, American Mathematical Society, 2005.

This three-volume set addresses the interplay between topology, functions, geometry, and algebra. Bringing the beauty and fun of mathematics to the classroom, the authors offer serious mathematics in a lively, reader-friendly style. Included are exercises and many figures illustrating the main concepts. It is suitable for advanced high-school students, graduate students, and researchers.