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★**An introduction to the representation theory of groups.**
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The book under review provides an introduction to the theory of group representations, with a non-exclusive focus on finite and compact groups. There are very few prerequisites: the objects and the formalism are gently introduced and the reader is guided by means of down-to-earth exercises and concrete examples through the basics of the theory so that the more abstract concepts seem natural when they are presented.

While intended for readers with little background, the book maintains the ambition to offer a broad view of a classical core of representation theory as well as an introduction to modern problems. It is in our opinion very successful in this regard. The treatment of basic generalities about various types of representations, elementary representation theory of finite groups and Peter-Weyl theory is accessible and nicely illustrated. The exposition of these classical topics is complemented by a number of thoughtfully chosen applications, resulting in a well-balanced ensemble. The more advanced topics include a discussion of the quantum numbers of the hydrogen atom and the Larsen alternative for unitary groups as well as some of its applications.

Other important areas of the theory are not presented in detail, but simply touched upon. This is in particular the case for the unitary representation theory of non-compact Lie groups. Once again, given the targeted audience, the author does an excellent job of indicating what the natural questions are when attempting to generalize Fourier analysis to non-compact Lie groups.

The author's philosophy is perhaps best illustrated by the following example: after a rather pedestrian treatment of Frobenius Reciprocity and some of its consequences, the notion of functoriality is introduced and presented as a way to alleviate the 'death of a thousand checks' inflicted upon the reader by the initial approach. To quote the author again, 'in writing this text, the objective has never been to give the shortest or slickest proof [. . .], rather to explain the ideas and the mechanism of thought that can lead to an understanding of "why" something is true, and not simply to the quickest line-by-line check that it holds.' And indeed, in many occasions throughout the book, preference is given to enlightening demonstrations while quicker or more sophisticated ones are left as exercises.

This book is a valuable contribution to the literature of a well-established field that certainly does not lack monumental treatises and formidable monographs, in the sense that it can be safely put in the hands of undergraduates with little mathematical baggage. Interested readers will be able to acquire a sense of the techniques, ideas, beauty and applications of representation theory while working their way from basics to topics of current interest.

Here is an outline of the book. In Chapter I, famous results with simple formulation related to prime numbers, quantum physics, word problems and finite groups are described, none of which are stated in representation-theoretic terms, to serve as motivation for the study of the subject. Chapter II, which takes up a quarter of the book, introduces the language and basic machinery of representation theory. It includes a treatment of Frobenius Reciprocity, the Jordan-Hölder-Noether Theorem, Burnside's irreducibility criterion and Schur's Lemma, as well as a number of detailed examples with different flavors, from finite groups to Lie groups. In Chapter III, the framework is extended beyond the strict algebraic notion of group representation. Basic facts are presented about group algebras, Lie algebras, topological groups and unitary representations.

Chapters IV and V are devoted to finite and compact groups respectively and constitute the heart of the book. The study of characters and Peter-Weyl Theory is carried out in a clear fashion and, once again, illustrated by a wealth of interesting examples. More advanced applications are considered in Chapter VI, including the Frobenius-Schur indicator, the Larsen alternative and quantum-theoretic facts related to representations of $\mathrm{SO}(3,\mathbb{R})$. Finally, Chapter VII introduces generalizations of the harmonic analysis discussed in the previous chapter to the non-compact case. Fourier analysis on locally compact abelian groups is briefly treated and some facts about infinite-dimensional representations of $\mathrm{SL}(2,\mathbb{R})$ are presented as an introduction to general non-commutative harmonic analysis.

A short appendix contains basic facts about algebraic integers, the Spectral Theorem and the Stone-Weierstrass Theorem. *Pierre Clare*