上 海 交 通 大 学 试 卷 (A卷)

(2017至2018 学年 第2学期 2018年6月14日)

班级号003-(2017-2018-2)MS107 学号______________

课程名称____概率论____ 姓名______________ 成绩______________

1. (10 分) A fair coin is tossed 100 times independently. Let H_{50} be the number of heads up in the first 50 tosses, and let H_{100} be the total number of heads up in the 100 tosses. Find the correlation coefficient between H_{50} and H_{100}.

我承诺，我将严格遵守考试纪律。
承诺人:__________

题号	一	二	三	四	五	六	七	八	九	十	总分
得分											

2. (10 分) Show that the geometric distribution is the only discrete distribution $\mathfrak{D}$ on the set of nonnegative integers with the property $P(X-k=m \mid X \geq k)=P(X=m)$ for all nonnegative integers k and m where $X \approx \mathfrak{D}$.

3. (10 分) Suppose X and Y are two random variables whose values are in $\{1, 2, \ldots, 9\}$. Suppose for all $i, j \in \{1, 2, \ldots, 9\}$ we have $P((X \geq i) \cap (Y \geq j)) \leq P(X \geq i) \cdot P(Y \geq j)$. Show that X and Y cannot be positively correlated.

4. (10 分) Let $X_1, X_2, \ldots, X_5$ be independent random variables which take values in $\{1, 2, 3\}$ with equal probability. Let $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(5)}$ be the non-decreasing ordering of $X_1, X_2, \ldots, X_5$. Compute $\mathbb{E}[X_{(3)} - X_{(2)}]$, $\mathbb{E}[X_{(4)} - X_{(3)}]$ and $\mathbb{E}[X_{(4)} - X_{(2)}]$.

5. (4 分) Are the following statements TRUE or FALSE?

- Let X, Y and Z be three random variables for which $\mathbb{E}(X+Y+Z)$ exists. Then we can determine the expectation $\mathbb{E}(X+Y+Z)$ from the individual distributions of X, Y and Z.

  ☐ TRUE ☐ FALSE

- Two points A and B are picked independently at random inside a disk C, namely hitting each region with a probability proportional to its area. Then the probability that the disk having centre A and radius $|AB|$ lies inside C is $\frac{1}{6}$.

  ☐ TRUE ☐ FALSE

6. (10 分) Let X be a random variable and let h be a function from $\mathbb{R}$ to $[0,2]$ such that $\mathbb{E}[h(X)]$ exists. Show that $P(h(X) \geq 1) \geq \mathbb{E}[h(X)] - 1$.

7. (10 分) There are n events, and the probability of each event is a constant $\epsilon > 0$. Show that if n is large enough, there exist three events among the given n events such that the probability that they occur simultaneously is positive.

8. (10 分) Consider two independent random walks on the cycle graph of length 20. At the beginning, two guys stand on a pair of antipodal vertices, that is to say, their distance in the cycle graph is 10. Every morning, each of them moves randomly to one of the two adjacent vertices in the cycle. Compute the expectation time of their first meeting on the cycle and the expectation time to meet again after their first meeting.

9. (10 分) Suppose that X and Y are independent random variables where X is exponentially distributed with expected value $\frac{1}{\alpha}$ and Y is exponentially distributed with expected value $\frac{1}{\beta}$. Prove that $P(X < Y) = \frac{\alpha}{\alpha+\beta}$.

10. (16 分)

10.a **(4pts).** State the definition of being pairwise independent and being mutually independent for events and random variables.

10.b **(4pts).** For the weak law of large numbers, can we use pairwise independence as the independence assumption?

10.c **(4pts).** Construct five random events which are pairwise independent but not mutually independent.

10.d **(4pts).** Let $U_1, \ldots, U_n$ be independent uniform $[0,1]$ random variables. If n is large, $(U_1 \cdots U_n)^{\frac{1}{n}}$ is most likely to be very close to a certain number g. Explain why and determine this number g.