MA193
2017 Summer Seminar in Mathematics (夏季数学讲座)
Zhongyuan 305 (中院305)

> Dare to know! Have the courage to use your own intelligence! – Immanuel Kant, Answering the Question: What Is Enlightenment? 1784.
> http://www.columbia.edu/acis/ets/CCREAD/etscc/kant.html
> http://gutenberg.spiegel.de/buch/-3505/1

June 27, 12:00–13:40, Youlin Li (李友林)

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Knots
纽结

A knot is an embedding $f:\ S^1 \to \mathbb{R}^3$ of a circle into $\mathbb{R}^3$. We will talk about Jones polynomials of knots and the applications of knot theory in biology and chemistry.

参考文献

June 27, 14:00–15:40, Jiajun Ma (马家骏)

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An invitation to the classical invariant theory (CIT): I
浅谈经典不变量理论（一）

After Hermann Weyl's celebrated book "The Classical Groups: Their Invariants and Representations" (1939), Classical Invariant Theory becomes an extensively studied subject and powerful tool in both mathematics and physics. Meanwhile, Weyl's book is not an easy reading. We will use a modern approach to the theory. Some explicit examples will be given to illustrate the theory.

In the first lecture, I will discuss the classical invariant theory for the general linear group. Along the discussion, the notation of group, (polynomial) algebra, representation and affine scheme (don't be afraid of the name) etc. will be briefly explained. Then I will state the first and second fundamental theorem of the classical invariant theory in both algebraic and geometric ways.

参考文献

[1] Roe Goodman, Nolan R. Wallach. Symmetry, Representations, and Invariants. *Springer, Dordrecht,* 2009.

[2] Roger Howe. Perspectives on invariant theory: Schur duality, multiplicity-free actions and beyond. *Bar-Ilan Univ., Ramat Gan, Israel*, 1995.

[3] Roger Howe. Remarks on classical invariant theory. *Trans. Amer. Math. Soc.*, 313:539–570, 1989.

[4] Roger Howe and Eng Chye Tan. Non-Abelian Harmonic Analysis: Applications of $\mathfrak{sl}(2)$. *Springer Science & Business Media*, 2012.

June 29, 14:00–15:40,   Jiajun Ma (马家骏)

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An invitation to the classical invariant theory (CIT): II

浅谈经典不变量理论（二）

After Hermann Weyl's celebrated book "The Classical Groups: Their Invariants and Representations" (1939), Classical Invariant Theory becomes an extensively studied subject and powerful tool in both mathematics and physics. Meanwhile, Weyl's book is not an easy reading. We will use a modern approach to the theory. Some explicit examples will be given to illustrate the theory.

In the first lecture, I will discuss the classical invariant theory for the general linear group. Along the discussion, the notation of group, (polynomial) algebra, representation and affine scheme (don't be afraid of the name) etc. will be briefly explained. Then I will state the first and second fundamental theorem of the classical invariant theory in both algebraic and geometric ways.

In the second lecture, we will discuss CIT for orthogonal and symplectic groups. The notion of formed spaces and the Witt's theorem will be given. Then I will present the first and second fundamental theorem for these groups. To state the second fundamental theorem of symplectic groups, Pfaffian will be introduced. If time permits, I will show how the joint actions of an $\mathfrak{sl}(2)$ and an orthogonal group on certain space of polynomials relates to the theory of spherical harmonics. From a representation theory point of view a reductive dual pair correspondence is behind.

参考文献

[1] Roe Goodman, Nolan R. Wallach. Symmetry, Representations, and Invariants. *Springer, Dordrecht*, 2009.

[2] Roger Howe. Perspectives on invariant theory: Schur duality, multiplicity-free actions and beyond. *Bar-Ilan Univ., Ramat Gan, Israel*, 1995.

[3] Roger Howe. Remarks on classical invariant theory. *Trans. Amer. Math. Soc.*, 313:539–570, 1989.

[4] Roger Howe and Eng Chye Tan. Non-Abelian Harmonic Analysis: Applications of $\mathfrak{sl}(2)$. *Springer Science & Business Media*, 2012.

June 29, 16:00–17:40, Zeng-Qi Wang (王增琦)

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On the matrix decompositions
矩阵分解

Can you pick up the most 'important' vectors from a set of vectors? How shall you do this? How shall you do if the numbers of vectors in the set is terribly large? We are going to answer the questions with the powerful tools – Matrix decompositions. I will show you some fundamental conception about LU, QR and SVD. Then, we move on to the rank revealing decompositions and their fashionable applications. You will find also that the design of the decomposition is an art.

参考文献

July 4, 14:00–15:40, Zeng-Qi Wang (王增琦)

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The way to design an efficient iteration methods–matrix splitting
矩阵分裂：设计高效迭代法的法门

I will show you how to design an iteration methods from a splitting of the coefficient matrix, and how the eigenvalues influence the convergence of the iteration methods. The preconditioning technique, as an essential tool to deal with the ill-conditioned problem, will be introduced also.

参考文献

July 4, 16:00–17:40, Yaokun Wu (吴耀琨)

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What are oriented matroids?
什么是定向拟阵？

Matroids were introduced by Hassler Whitney in 1935, partly due to his effort in tackling the four-color conjecture. Matroids capture the the essence of the concept of "independence" and turn out to be a very basic structure in a vast range of mathematics and sciences [7, 12]. If you have heard of differential geometry and algebraic geometry, you may like to know that Gian-Carlo Rota called a simple matroid a combinatorial geometry. The study of matroids offers students a unique opportunity to synthesize several different areas within mathematics typically studied at the undergraduate level [12, Preface]. Going beyond matroids, mathematicians have subsequently invented greedoids, oriented matroids (sometimes also called "chirotopes" by stereochemists), valuated matroids, Coxeter matroids, arithmetic matroids, complex, quarternionic, and octonionic matroids, matroids with coefficients, and so on [1, 2, 3, 5, 6, 8, 9, 10, 11, 13, 14, 15].

In matroid theory, one replaces the value of a determinant by a bit of information whether it is zero or nonzero; in oriented matroid theory, one replaces the value of a determinant by its sign, which is a trit of information. The preface of [3] starts with the following description: "Oriented matroids are a very natural mathematical concept which presents itself to us in many different guises, and which has connections and applications to many different areas. These areas include discrete and computational geometry, combinatorics, convexity, topology, algebraic geometry, operations research, computer science, and theoretical chemistry."

In this talk, we will use concrete easy examples, which you should have seen many times before, to demonstrate to you how some oriented matroid structures are hidden in some guises there [2, 3, 4]. We try to show that, once realizing that underlying oriented matroid structure, you can still do "linear algebra" without having any linear space.

参考文献

[1] Michele D'Adderio, Luca Moci. Arithmetic matroids, the Tutte polynomial and toric arrangements. *Advances in Mathematics*, 232 (2013) 335-367.

[2] Achim Bachem, Walter Kern, Linear Programming Duality: An Introduction to Oriented Matroids. Springer-Verlag, Berlin, 1992.

[3] Anders Björner, Michel Las Vergnas, Bernd Sturmfels, Neil White, Günter M. Ziegler. Oriented Matroids. *Encyclopedia of Mathematics*, Vol. 46, Cambridge University Press, 1993.

[4] Jürgen G. Bokowski. Computational Oriented Matroids: Equivalence classes of matrices within a natural framework. Cambridge University Press, Cambridge, 2006.

[5] Alexandre V. Borovik, Israel M. Gelfand, Neil White. Coxeter Matroids. *Progress in Mathematics*, Vol. 216, Birkhäuser, 2003.

[6] Johannes Brunnemann, David Rideout. Oriented matroids–combinatorial structures underlying loop quantum gravity. *Classical and Quantum Gravity*, 27 (2010) 205008.

[7] William H. Cunningham. The coming of the matroids. *Documenta Mathematica*, (2012) 143–153.

[8] Andreas Dress, Andre S. Drieding, Hans Rudolf Haegi. Classification of mobile molecules by category theory. In: Jean Maruani and Josiane Serre (eds.) Symmetries and Properties of Non-Rigid Molecules: A Comprehensive Study, *Studies in Physical and Theoretical Chemistry*, vol. 23, pp. 39–58, Elsevier Scientific, 1983.

[9] Andreas W.M. Dress. Duality theory for finite and infinite matroids with coefficients. *Advances in Mathematics*, 59 (1986) 97–123.

[10] Andreas W.M. Dress, Walter Wenzel. Arithmetic and polynomials over fuzzy rings. *Communications in Algebra*, 43 (2015) 1207–1231.

[11] Alex Fink, Luca Moci. Matroids over a ring. *Journal of the European Mathematical Society*, 18 (2016) 681–731.

[12] Gary Gordon, Jennifer McNulty. Matroids: A Geometric Introduction. Cambridge University Press, 2012.

[13] Tamás Hausel. Quaternionic geometry of matroids. *Central European Journal of Mathematics*, 43 (2005) 1207–1231.

[14] Bernhard Korte, László Lovász, Rainer Schrader. Greedoids. *Algorithms and Combinatorics*, Vol. 4, Springer, 1991.

[15] Kazuo Murota. Systems Analysis by Graphs and Matroids. *Algorithms and Combinatorics*, Vol. 3, Springer, 1987.

July 6, 14:00–15:40, Weike Wang (王维克)

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The structure of mathematical spaces
数学空间的结构

Starting from the concepts of linear space and metric space, we introduce the generalized concepts of mathematical spaces. We will focus on mathematical spaces with linear structures and topological structures and address their applications in advanced analysis.

参考文献

July 6, 16:00–17:40, Mijia Lai (来米加)

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Alexsandrov Theorem: A short introduction to geometric analysis
Alexsandrov定理：几何分析简介

A celebrated theorem of Alexsandrov asserts that an embeded closed hypersurface with constant mean curvature must be a round sphere. In this talk, I shall first introduce the rudiments of differential geometry and partial differential equations; then give a presentation of Alexsandrov's proof: the method of moving plane. The theorem is a nice illustration of the subject of geometric analysis: solving geometric problems with analytical tools.

参考文献

July 11, 14:00–15:40, Cheng Wang (王成)

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High dimensional probability

大数据中的概率

In this talk, I will introduce some basic concentration inequalities which are useful for high dimensional data. Specially, I will present the Stirling's formula and its application in the law of large numbers, the Hoeffding's inequality and the Chernoff's inequality. Two applications will also be discussed.

参考文献

[1] Vershynin, Roman. High Dimensional Probability. 2016.

[2] Tao, Terence. Topics in Random Matrix Theory. 2012.

[3] 陈希孺. 数理统计学简史. 2002.

July 11, 16:00–17:40, Jia-Rui Fei (费佳睿)

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Laurent phenomenon: I

洛朗奇迹（一）

A composition of birational maps given by Laurent polynomials need not be given by Laurent polynomials; however, sometimes—quite unexpectedly—it does. We explore such phenomenon in many innocent situations. We state Fomin and Zelevinsky's sufficient conditions. We explain how this related to the notion of the quiver mutation.

In this first lecture, I will introduce Laurent phenomenon through some interesting number theory problem. I will play some classical examples (eg. Somos sequence) and many new examples (never published). I hope to attract students to the Diophantine problems, which constantly motivate the development of modern mathematics. I may spend half of the lecture digressing on the classical problems of quadratic and elliptic curves. In the end, I will reveal the algebra behind them.

In the second lecture, we will focus on algebras rather than numbers. I will formalize the Laurent phenomenon, and introduce one or two theorems of Fomin and Zelevinsky. I will show some other occurrence of the phenomenon in mathematics. Students can see some representation theory. I try to explore the systematic methods to produce such phenomenon. Among them the quiver mutation is the most developed one. If time permit, I will introduce the cluster algebra.

Throughout two lectures I will pose exercises and open problems.

参考文献

[1] S. Fomin and A. Zelevinsky. The Laurent phenomenon. *Adv. in Applied Math.* 28 (2002), 119–144.

[2] S. Fomin and A. Zelevinsky. Cluster algebras I: Foundations. *J. Amer. Math. Soc.* 15 (2002), 497–529.

[3] D. Gale. The strange and surprising saga of the Somos sequences. *Math. Intelligencer 13 (1991),* no. 1, 40–43.

July 13, 14:00–15:40, Feng Rong (戎锋)

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Schwarz Lemma: Analysis and geometry

施瓦兹引理：分析与几何

Schwarz Lemma is one of the most fundamental results in complex analysis. We give an introduction of it from both the analytic and geometric view point, and also some of its generalizations.

参考文献

July 13, 16:00–17:40, Jia-Rui Fei (费佳睿)

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Laurent phenomenon: II

洛朗奇迹（二）

A composition of birational maps given by Laurent polynomials need not be given by Laurent polynomials; however, sometimes—quite unexpectedly—it does. We explore such phenomenon in many innocent situations. We state Fomin and Zelevinsky's sufficient conditions. We explain how this related to the notion of the quiver mutation.

In this first lecture, I will introduce Laurent phenomenon through some interesting number theory problem. I will play some classical examples (eg. Somos sequence) and many new examples (never published). I hope to attract students to the Diophantine problems, which constantly motivate the development of modern mathematics. I may spend half of the lecture digressing on the classical problems of quadratic and elliptic curves. In the end, I will reveal the algebra behind them.

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[2] S. Fomin and A. Zelevinsky. Cluster algebras I: Foundations. *J. Amer. Math. Soc.* 15 (2002), 497–529.

[3] D. Gale. The strange and surprising saga of the Somos sequences. *Math. Intelligencer 13 (1991),* no. 1, 40–43.

July 18, 14:00–15:40, Feng Xie (谢峰)

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Fixed point theorems and their applications
不动点定理及其应用

In this talk we will talk about several fixed point theorems and their applications. In particular, we will discuss their applications in solving equations.

参考文献

July 18, 16:00–17:40, Fang Wang (王芳)

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Propagation of singularities
奇性传播

In this lecture, I will first introduce the wave front set to describe the singularities of a function on Euclidean spaces or manifolds. Then I will discuss how the singularities of solutions to a linear PDE propagate along the Hamilton flow defined by the principle symbol of the linear operator (Hormander's theorem). If time permits, the application to some nonlinear PDEs will also be given.

参考文献

July 20, 14:00–15:40, Xiaoqun Zhang (张小群)

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Mathematics in imaging and data sciences

影像科学与数据科学中的数学

Image progressing and analysis, computer vision, image recognition are terms that refer to the process of modeling and computing with images. This process has been made possible by the advent of computers powerful enough to cope with the large dimensionality of image data and the complexity of the algorithms that operate on them. If we now wonder as to the mathematics relevant to imaging and data sciences, we can come up with a surprisingly long list: Differential and Riemannian geometry, functional analysis (calculus of variations and partial differential equations), probability theory (probabilistic inference, Bayesian probability theory), statistics, computational harmonic analysis, optimization and singularity theory etc.

In this lecture, I will introduce some mathematical concepts, models and algorithms that are successfully employed in this field, with diverse applications to biomedical imaging, entertainment industry, engineering, and geophysics etc.

参考文献

July 20, 16:00–17:40, Yihu Yang (杨义虎)

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从欧几里得到罗巴切夫斯基 – 漫谈非欧几何

From Euclid to Lobachevsky: An introduction to non-Euclidean geometry

在这个演讲里，我将描述非欧几何的发展历史以及罗巴切夫斯基非欧几何的基本思想；在此过程中，我也将描述高斯关于3-维欧式空间中曲面几何的基本工作以及庞加莱的非欧几何实现。进一步地，我也将结合高斯的工作来描述黎曼几何的一些基本概念。

参考文献
