Course description

This course aims to provide a tasty appetizer for students to appreciate and play with some wonderful mathematics. We will address a series of different topics, which are **easily digested** and may point to some challenging directions. In the course of studying together with the students the work and story of some interesting mathematicians, we try to show the richness and also the unity of mathematics at an elementary level. Though for each topic we may only have time to have a glimpse of it, we expect to give the students an eye-opening experience so that

- some of them can be motivated to start their own mathematical route; and
- most of them can at least know that mathematics is an art and, with a little training, they can have lots of fun from mathematics in their daily life, just like whatever they may have from sports, music or foods.

As an additional hook that entices the audience, we will from time to time prepare some research problems which are relevant to the topics discussed in the classroom.

The final score of the students registered to this course will be judged by a presentation/report on any topics related to whatever discussed in this course and this presentation/report can be based on several research papers or books or the researches of the students.

Here is a tentative list of the possible mathematics topics to be addressed in this course.

- Catalan number
- Tree reconstruction
- The Amitsur-Levitzki Theorem
- Manjul Bhargava, The Factorial Function and Generalizations, American Mathematical Monthly 107 (2000) 783–799.
- Quadratic reciprocity and Zolotarev's Lemma
- The modular space of all lines in the plane is the open Möbius band; Classification of surfaces.
- EKR Theorem
- Morphisms and discrete isoperimetric problems
- Factorization of Abelian groups
- Lecture 3: On Collecting Like Terms, on Euler, Gauss and MacDonald, and on Missed Opportunities, in: Dmitry Fuchs (`https://www.math.ucdavis.edu/research/profiles/?fac_id=fuchs`), Sergei Tabachnikov (`http://www.math.psu.edu/tabachni/`), Mathematical Omnibus: Thirty Lectures on Classic Mathematics (`http://www.math.psu.edu/tabachni/Books/taba.pdf`), AMS, 2007.

- The Robinson-Schensted-Knuth correspondence
- Interval graphs; First-Fit coloring
- Quadratic form
- Dollars game and Riemann-Roch Theory for graphs
- Treewidth and the robbers game
- Electrical network
- The Banach-Tarski Paradox
- The Alexander number and the Alexander group of a knot
- Oriented matroid
- Hyperplane arrangement
- De Bruijn - Erdös Theorem
- The Szemerédi-Trotter incidence Theorem
- Snevily's Conjecture (`http://www.cs.elte.hu/~karolyi/snevily6.pdf`)

Here are some books/websites which the participants of this course might like to have a look at.

1. Achim Bachem (http://www.wincor-nixdorf.com/internet/cae/servlet/contentblob/684086/publicationFile/64940/Bachem.pdf), Walter Kern, Linear Programming Duality: An Introduction to Oriented Matroids, Universitext, Springer, 1992.

2. Alexandre V. Borovik (http://www.maths.manchester.ac.uk/~avb/), Mathematics under the Microscope: Notes on Cognitive Aspects of Mathematical Practice, AMS, 2009.

3. John Horton Conway (http://www.math.princeton.edu/directory/john-conway), The Sensual (quadratic) Form, Carus Mathematical Monographs (Book 26), MAA, 2005.

4. Peter G. Doyle (https://www.math.dartmouth.edu/~doyle/), Laurie Snell (http://www.dartmouth.edu/~chance/jlsnell.html), Random Walks and Electric Networks, Carus Mathematical Monographs (Book 22), MAA, 1984.

5. William Fulton (http://www.math.lsa.umich.edu/~wfulton/), Young Tableaux, With Applications to Representation Theory and Geometry, London Mathematical Society Student Texts 35, Cambridge University Press, 1997.

6. Jean Gallier (http://www.cis.upenn.edu/~jean/), Dianna Xu (http://cs.brynmawr.edu/~dxu/), A Guide to the Classification Theorem for Compact Surfaces (http://www.cis.upenn.edu/~jean/surfclass-n.pdf), Springer, 2013.

7. Julia Garibaldi, Alex Iosevich (http://www.math.rochester.edu/people/faculty/iosevich/), Steven Senger, The Erdös Distance Problem, Student Mathematical Library, Vol. 56, AMS, 2010.

8. Thomas Garrity, Richard Belshoff, Lynette Boos, Ryan Brown, Carl Lienert, David Murphy, Junalyn Navarra-Madsen, Pedro Poitevin, Shawn Robinson, Brian Snyder (http://math.lssu.edu/bsnyder/PCMI/), Caryn Werner, Algebraic Geometry: A Problem Solving Approach (http://www.ams.org/bookstore-getitem/item=stml-66), Student Mathematical Library, Vol. 66, AMS, 2013.

9. Lawrence H. Harper (http://www.math.ucr.edu/~harper/), Global Methods for Combinatorial Isoperimetric Problems, Cambridge studies in advanced mathematics, Vol. 90, Cambridge University Press, 2004.

10. Stasys Jukna (http://lovelace.thi.informatik.uni-frankfurt.de/~jukna/index.html), Extremal Combinatorics: With Applications in Computer Science (http://lovelace.thi.informatik.uni-frankfurt.de/~jukna/EC_Book_2nd/draft.pdf), 2nd Edition, Springer, 2011.

11. Miklós Laczkovich, Conjecture and Proof, Classroom Resource Materials, MAA, 2001.

12. Ezra Miller (`http://www.math.duke.edu/~ezra/`), Victor Reiner (`http://www-users.math.umn.edu/~reiner/`), Bernd Sturmfels (`http://math.berkeley.edu/~bernd/`), Eds., Geometric Combinatorics, IAS/Park City Mathematics Series, AMS, 2007.

13. Sherman Stein (`https://www.math.ucdavis.edu/research/profiles/?fac_id=stein`), Sandor Szabó, Algebra and Tiling: Homomorphisms in the Service of Geometry, Carus Mathematical Monographs (Book 25), MAA, 2010.

What is really going on?

We will expect that some students can help to write a notes for each lecture. The notes should give more ideas on what is going on in the classroom. The students may also like to insert some more thoughts on the topic there in their notes.

We record/announce the rough content of the lecture in each week here. We usually make the announcement one or two weeks earlier but may update it later subject to any possible change/addition of the topics. We may often just list some relevant papers/books. We either make use of some material there in our teaching or we recommend (but NOT require) the students read it before/after the class.

Sep. 16

- Doron Zeilberger, $\binom{5}{2}$ Proofs that $\binom{n}{k} \leq \binom{n}{k+1}$ if $k < n/2$, 2010, `http://arxiv.org/pdf/1003.1273.pdf`
- `http://www.math.rutgers.edu/~zeilberg/Opinion104.html`
- Michael Dairyko, Claudia Rodriguez, Schuyler Veeneman, Bijection from Shi arrangement regions to parking functions via mixed graphs, 2012, `http://math.sfsu.edu/beck/teach/msriup/mike.claudia.sky.pdf`
- Sam Hopkins, David Perkinson, Orientations, semiorders, arrangements, and parking functions, The Electronic Journal of Combinatorics 19 (2012) Paper #P8, 31 pp `http://www.combinatorics.org/ojs/index.php/eljc/article/view/v19i4p8`.

- Symmetric (semisymmetric) chain decompositions of Boolean algebra, partition lattice, the lattice of noncrossing partitions
- Bijections among parking function, trees, noncrossing partitions, hyperplane arrangements, Dyck paths...

Sep. 23

- Jean-Louis Loday, Dichotomy of the addition of natural numbers, 2011, `http://arxiv.org/pdf/1108.6238.pdf`
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