

Honor Policy. You **should not** simply copy the answer of your classmates. You can make use of the library or any resources from the internet. The points earned from each problem will depend on the average quality of the answers to that one from all students.

Directions. Please type your answers in \LaTeX . You could write in English or in Chinese. Please guarantee that your text will be easy to understand. Please print out the answer sheet, sign your name and submit it to Zeying Xu (徐泽瀛) before May 12, 2014.

Name: [你的名字, 电脑输入和手工签名]

Student Id: [你的学号]

If you use any resources, say a hint from a professor, a discussion with some fellow students, a book, a published paper, a lecture notes or some answers from a math forum, please record them here for our reference:

Problem 1: Let $X = \{a, b, c, d, e\}$ and let $D : \binom{X}{3} \rightarrow \mathbb{R}$ be the map such that $D(\{a, b, c\}) = 4, D(\{a, b, e\}) = D(\{a, c, e\}) = D(\{b, c, e\}) = D(\{c, d, e\}) = 6, D(\{b, d, e\}) = D(\{b, c, d\}) = D(\{a, d, e\}) = D(\{a, b, d\}) = 7, D(\{a, c, d\}) = 8$. Let $P = \{f \in \mathbb{R}^X : f(x) + f(y) + f(z) \geq D(\{x, y, z\}) \text{ for all } \{x, y, z\} \in \binom{X}{3}\}$. Determine the set of those minimal elements in P .

Answer: Your answer here

Problem 2: Let D be a symmetric bivariate map on a set X . Show that $(T(D), L_\infty^D)$ is Δ -hyperbolic provided (X, D) is Δ -hyperbolic.

Answer: Your answer here

Problem 3: Let X be a finite set of size n . We say that a quartet $ij \mid kl$ is contained in a split $\{A, X \setminus A\}$ provided $|\{i, j\} \cap A| \in \{0, 2\}$ and $|\{i, j, k, \ell\} \cap A| = 2$. We assume that Σ is a split system on X such that, for every $\{i, j, k, \ell\} \in \binom{X}{4}$, at most two of the three quartets $ij \mid kl, ik \mid j\ell$, and $i\ell \mid kj$ can be contained in some splits from Σ . Please estimate the size of Σ . Especially, please determine the maximum size of such a split system Σ for several special values of n .

Answer: Your answer here

Problem 4: A quartet system Q on a finite set X is compatible provided it is contained in the quartet system of a phylogenetic X -tree. Try to construct a quartet system Q on a set of seven elements such that Q is not compatible but every proper subset of Q is compatible. Estimate the largest size of such a quartet system Q . Could you also say something when X has eight elements?

Answer: Your answer here

Problem 5: Let G be a graph on the vertex set V and edge set E . We define the map of G , denoted f_G , to be the binary linear map from $2^V / \{\emptyset, V\}$ to $P_{\text{even}}(V)$ such that $f_G(\{A, X \setminus A\})$ is the set of odd degree vertices in the graph $(V, E \setminus ((\binom{A}{2} \cup \binom{B}{2})))$. Show that the graph G has an odd number of spanning trees if and only if f_G is a linear isomorphism. Estimate the maximum number of trees on V such that their maps are linearly independent over the binary field.

Answer: Your answer here

Problem 6: Let T be a binary phylogenetic X -tree with positive edge weights. Choose $A \subseteq \binom{X}{3}$ such that med_T gives a bijection from A to the set of inner vertices of T . Show that the weighted tree T can be uniquely reconstructed from A and the sets $\{D_T(a_x, a_y), D_T(a_y, a_z), D_T(a_z, a_x)\}$ for all $\{a_x, a_y, a_z\} \in A$.

Answer: Your answer here

Problem 7: Let X be a finite set and let Σ be a split system over X such that the incomparability graph G of Σ is connected. The set of all cliques of G under the inclusion relationship is a poset P . Show that the Buneman graph of G is isomorphic to the Hasse diagram of P .

Answer: Your answer here

Problem 8: Let X be a finite set and take $P \subseteq \binom{X}{2}$. We say that P labels a phylogenetic X -tree T if there is a bijection between P and $E(T)$ such that the edge $e \in E(T)$ corresponding to $\{x, y\} \in P$ lies in the unique path in T connecting x and y . Please provide some necessary or/and sufficient conditions for those sets $P \subseteq \binom{X}{2}$ which can label some phylogenetic X -tree.

Answer: Your answer here

Problem 9: If you have prepared some lecture notes, solved some problems mentioned in my lectures, had some observations or conjectures related to something discussed in my course, or made a reading report on some material on phylogenetic combinatorics, and so on, you are invited to present them or list them below for our reference. It may help us understand your progress in this course and help to improve your scores.

Answer: Your presentation here