Midterm Examination

MA156 Symbolic Dynamics (Fall 2013)

18: 00 – 20: 20, Nov. 21, 2013, Xia Yuan 106

Name: ____________________

Student ID: ____________________

Score: ____________________

Instructions

1. Remember the following quote of Steve Jobs (1955 - 2011): "Quality is more important than quantity. One home run is much better than two doubles."

2. Before you submit your answer sheet, please give an ordering of the 9 problems here:

   Problem __≥ Problem __≥ Problem __≥ Problem __≥ Problem __≥ Problem __≥ Problem __≥ Problem __≥ Problem __.

   We will check your solutions according to this linear ordering. After grading the solutions to your first m problems, we may stop reading your remaining solutions, if any, where m is the smallest number no less than 5 such that you fail to get half of the total scores for your first m problems or is simply 9 when such a number does not exist. If you do not give a specific ordering above, we will understand that the ordering is the natural one from 1 to 9.

3. Please say what you mean and mean what you say!

4. When writing down your answer, you are not expected to just put there what you thought to be a correct one, you are expected to please us with your writing, let us understand your argument easily and let us appreciate your presentation. When you cannot give a complete solution to one problem, we may give you considerably partial credit if you can clearly and honestly describe a plan of your solution and what are the missing steps in your proof. It is better to give nothing than to produce a load of rubbish.

5. You may consult the textbook, your notes, or any other books during the examination. You are not allowed to use the internet and you should not work with anyone else on the exam. It is better to give nothing than to cheat.

6. Your score for this midterm exam will be submitted to the teaching affairs office after some order-preserving transformation (which is still unknown at present).

Good luck!

Problem 1 (15 points) Let X be a shift space with only one follower set. Show that X is a full shift.

Problem 2 (15 points) Consider the sliding block code $\phi : \{0,1\}^{\mathbb{Z}} \to \{0,1\}^{\mathbb{Z}}$ defined by the rule $\phi(x)_i = x_i + x_{i+1} \pmod 2$. Let n be a positive integer and let m be the multiplicity of 2 in the prime factorization of n, namely m is the largest exponent for which 2^m divides n exactly. Show that the number of fixed points of ϕ^n is 2^{n-2^m}.

Problem 3 (15 points) Let $\mathfrak{B} : 2^{\mathcal{A}^{\mathbb{Z}}} \to \cup_{n=0}^{\infty} \mathcal{A}^n$ be the map which sends a subset X of the full shift to its language $\mathfrak{B}(X)$. Define $X \approx Y$ if $\mathfrak{B}(X) = \mathfrak{B}(Y)$. Show that $\approx$ is an equivalence relation and each equivalence class for $\approx$ contains exactly one shift space over $\mathcal{A}$.

Problem 4 (15 points) Let $\mathcal{G}$ be the minimal right-resolving presentation of an irreducible sofic shift X, and let r be the number of vertices of $\mathcal{G}$. Show that if X has finite type, then it actually must be $\frac{r^2-r}{2}$-step.

Problem 5 (15 points) Find the minimal right-resolving presentation for the sofic shift presented by

$$\begin{bmatrix} a & \emptyset & c & b \\ a & c & \emptyset & b \\ a & c & \emptyset & b \\ c & a & \emptyset & b \end{bmatrix}.$$

Problem 6 (15 points) Let $\Phi : \{0,1\}^3 \to \{0,1\}$ be the map given by $\Phi^{-1}(1) = \{011, 100\}$. Show that Φ_∞ is an onto map but not a conjugacy from the even shift to the golden mean shift.

Problem 7 (15 points) Let G be the graph with vertex set $\{v_1, \ldots, v_6\}$ and arc set $\{\overrightarrow{v_1v_2}, \overrightarrow{v_2v_3}, \overrightarrow{v_3v_1}, \overrightarrow{v_4v_5}, \overrightarrow{v_5v_6}, \overrightarrow{v_6v_4}, \overrightarrow{v_1v_5}, \overrightarrow{v_6v_3}\}$. Show that there is a graph H obtained from G by a sequence of out-splittings such that H has a vertex with exactly 3 outgoing arcs.

Problem 8 (15 points) Find a point mapping from the golden mean shift to itself that commutes with the shift, but is not a sliding block code.

Problem 9 (15 points) Show that for any two positive integers M and N, there exists an M-step shift of finite type which is conjugate to a shift space which is not N-step shift of finite type.