

图论

(官方课程表上一个媚俗的名称为：图与网络)

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*The area of mathematics that inspired most of my work is graph theory. This studies structures that are mathematically very simple: they consist of a some elements called nodes or vertices, and a relation between them: some pairs of nodes are adjacent, or connected by an edge, some pairs are not. Such a structure is called a **graph** or **network**. For example, if we want to describe the acquaintanceship structure of a society, we can represent every person by a node, and connect two nodes by an edge if these two persons know each other. We usually represent the nodes by points in the plane, and the edges, by curves connecting them (not necessarily straight). ... A graph is one of the simplest possible structures, and it is clear that graphs are needed for the representation or modeling of a great variety of structures in sciences, economics, engineering, logistics, and many other areas. – László Lovász, A fledgling subject bridging classical theory and new applications, Kyoto Prize Commemorative Lecture. 2010.*

A graph G is a pair of sets $V(G)$ and $E(G)$ such that $E(G) \subseteq \binom{V(G)}{2}$.

A hypergraph G is a pair of sets $V(G)$ and $E(G)$ such that $E(G) \subseteq 2^{V(G)}$.

A directed graph (digraph) G is a pair of sets $V(G)$ and $A(G)$ such that $A(G) \subseteq V(G) \times V(G)$.

What to ask about graphs? How to model/represent them?
How to approximate them? Are there any beautiful things about this seemingly very poor structure? Are they related to any other mathematical structures we encounter in university? Is there really a THEORY about graphs?

First Section of L. Lovasz, Very Large Graphs

1 Introduction

1.1 Huge networks

1.2 What to ask about them?

1.3 How to obtain information about them?

1.3.1 Local sampling

1.3.2 Observing global processes

1.3.3 Left and right homomorphisms

1.4 How to model them?

1.4.1 Random graphs

1.4.2 Randomly growing graphs

1.4.3 Quasirandom graphs

1.5 How to approximate them?

1.5.1 The distance of two graphs

1.5.2 Approximation by smaller: Regularity Lemma

1.5.3 Approximation by infinite: convergence and limits

1.5.4 Optimization problems for graphs

1.6 Mathematical tools

Some Random Problems I

How many graphs are there on n vertices?

What is the diameter of a typical graph on n vertices, n^2 , n ? $\frac{n}{2}$?
 \sqrt{n} ? $\log n$? 4? 2? 1?

How does a diameter 2 graph look alike? How does a connected graph look alike?

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Some Random Problems II

We say that a graph G has a closed (open) interval representation if each vertex of $V(G)$ can be associated a closed (open) interval I_v on the real line such that two different vertices u and v are adjacent in G if and only if $I_u \cap I_v \neq \emptyset$.

Is it true that G has an open interval representation if and only if it has a closed interval representation? Is it true when $V(G)$ is a finite set? What about the case that $V(G)$ is countably infinite or even uncountable?

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Some Random Problems III

Let k be a positive integer.

Is there any digraph such that for any two different vertices u and v there is a unique walk of length k from u to v and there is no walks of length k from v to itself?

Can you classify all such digraphs?

Some Random Problems III

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Some Random Problems IV

Let G be the cube graph.

Is it possible to associate to each of the 8 vertices u a vector f_u in the real Euclidean plane such that two different vertices u and v are adjacent if and only if the inner product of f_u and f_v is at least 1?

Some Random Problems IV

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Many eyes go through the meadow, but few see the flowers in it. – Ralph Waldo Emerson

Few are those who see with their own eyes and feel with their own hearts. – Albert Einstein

When it is dark enough, you can see the stars. – Ralph Waldo Emerson

By far the greatest impediment and aberration of the human understanding arises from [the fact that]...those things that strike the sense outweigh things which, although they may be more important, do not strike it directly. Hence, contemplation usually ceases with seeing, so much so that little or no attention is paid to things invisible. – Francis Bacon

The painter should not paint what he sees, but what will be seen. – Paul Valéry

The artist must create a spark before he can make a fire and before art is born, the artist must be ready to be consumed by the fire of his own creation. – Auguste Rodin

Nothing in life is to be feared, it is only to be understood. Now is the time to understand more, so that we may fear less. – Marie Curie

The greatest obstacle to discovery is not ignorance – it is the illusion of knowledge. – Daniel J. Boorstin

*Having vegetated on the fringes of mathematical science for centuries, combinatorics has now burgeoned into one of the **fastest growing branches of mathematics** . . . The mathematical world had been attracted by the success of algebra and analysis and only in recent years has it become clear . . . that **combinatorics, the study of finite sets and finite structures, has its own problems and principles.** These are independent of those in algebra and analysis but match them in **difficulty, practical and theoretical interest, and beauty.** – László Lovász, *Combinatorial Problems and Exercises, Second Edition, North Holland, 1993.**

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Gowers: "Problem Solvers" vs "Theory Developers"

If the processes of abstraction and generalization, which are so important in mathematics, are of only limited use in combinatorics, then how can the subject **be transmitted to future generations**? One way of thinking about this question is to ask what the requirements of tomorrow's combinatorialists are likely to be . . . their priority is likely to be solving problems, so their interest in one of today's results will be closely related to whether, by understanding it, they will improve their own problem-solving ability. And this brings us straight to the heart of the matter. **The important ideas of combinatorics do not usually appear in the form of precisely stated theorems, but more often as general principles of wide applicability.** – Timothy Gowers, *The Two Cultures of Mathematics*.

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根据经验法则解决问题这类的数学最明确最完全的例子就是组合学。不过也有部分组合学是理论建构的，并且有越来越多这样的例子出现。我想这是组合学为什么受欢迎或为数学家接受的一个原因，也就是以后组合学会有越来越多的结构跟理论。—《有朋自远方來—专访Jaroslav Nešetřil 教授》

Many Faces of Modern Combinatorics

As Anders Björner and Richard Stanley point out in [3], the reasons for the recent growth of combinatorics come from its relationship with areas at the **center stage of mathematics** on the one hand, and with **other disciplines (most of all computer science)**, on the other hand. This is a **two way relationship**: combinatorics has found important applications to other areas of pure mathematics and computer science, while certain mathematical techniques (for instance, from linear algebra, group theory, geometry, probability theory, statistical mechanics etc.) and computer experiments are now used in combinatorics.

– Cristian Lenart, The Many Faces of Modern Combinatorics.

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Interview of Bollobás

The usual charge against graph theory is, “Ah, it is made up of ad hoc problems that have nothing to do with each other. What’s the point?” To some extent, at the very beginning, this is true, but slowly, slowly all these results do gel into a single theory, so my aim was to show that there is such a theory ... – B. Bollobás

<http://www2.ims.nus.edu.sg/imprints/interviews/BelaBollobas.pdf>

Interview of Bollobás

For me, the difference between combinatorics and the rest of mathematics is that in combinatorics we are terribly keen to solve one particular problem by whatever means we can find. So if you can point us in the direction of a tool that may be used to attack a problem, we shall be delighted and grateful, and we'll try to use your tool. However, if there are no tools in sight then we don't give up but we'll try to use whatever we have access to: bare hands, ingenuity, and even the kitchen sink. Nevertheless, it is a big mistake to believe that in combinatorics we are against using tools - not at all. We much prefer to get help from "mainstream" mathematics rather than use "combinatorial" methods only, but this help is rarely forthcoming. However, I am happy to say that the landscape is changing. – B. Bollobás

Interview of Bollobás

The achievement is not in applying such a theorem, after all, every schoolboy knows the theorem, but in discovering that it can be applied, and how it can be applied ...

The trouble with the combinatorial problems is that they do not fit into the existing mathematical theories. They are not about functions, topological spaces, groups or operators. More often than not, we simply do not have the machinery to attack our problems. This is certainly not the situation in other branches of mathematics. In fact, it may happen that first a wonderful machine is built and then the search starts for a worth-while problem that this machine can be applied to. This attitude is totally foreign to combinatorics. – B. Bollobás

To be sure, it is very important to seek a deeper framework or paradigm of research, and it is always exciting if a problem or result from some seemingly remote area, all of a sudden fits in a nice abstract framework; often this reveals important points that have been missed and suggest interesting problems to work on. However, I have felt that results and methods that do not fit in these established frameworks are perhaps even more interesting. – L. Lovász, The 2010 Kyoto Prize Commemorative lectures.

《数学传播》刊登的两篇访谈

有朋自远方來——专访Gyula O. H. Katona教授,
<http://www.math.sinica.edu.tw/math-media/d301/30102.pdf>

有朋自远方來——专访Jaroslav Nesetril 教授,
<http://w3.math.sinica.edu.tw/math-media/d331/33101.pdf>

‘以管窥天，以蠡测海，以莛撞钟，岂能通其条贯，考其文理，发其音声哉！’ – 东方朔

The history of mathematics shows that “point of view” can be very important. What is difficult from one point of view may become easy from another. The classical Greek problems of constructing tangents for a plane curve and calculating the area enclosed by such curves were effectively solved only after the introduction of Cartesian coordinates. This allowed geometry to be translated into algebra, from which the patterns of the solutions sprang forth, creating calculus. Generally, the more varied and effective the points of view which a subject admits, the more profound and useful it becomes. – L.H. Harper, Preface for Global Methods for Combinatorial Isoperimetric Problems.

- ▶ C. Berge, Graphs and Hypergraphs, North-Holland, 1976.
- ▶ H.N.V. Temperley, Graph Theory and Applications, John Wiley, 1981.
- ▶ B. Bollobás, Combinatorics: Set Systems, Hypergraphs, Families of Vectors, and Combinatorial Probability, Cambridge University Press, 1986.
- ▶ G. L. Nemhauser, L. A. Wolsey, Integer and Combinatorial Optimization, Wiley, 1988.
- ▶ A.E. Brouwer, A.M. Cohen, A. Neumaier, Distance-Regular Graphs, Springer, 1989.
- ▶ R. Diestel, Graph Decompositions, Oxford University Press, 1990.
- ▶ L. Babai, P. Frankl, Linear algebra methods in combinatorics with applications to geometry and computer science, 1992, Comput. Sci. Dept., University of Chicago.
- ▶ N. Biggs, Algebraic Graph Theory, 2nd ed., Cambridge University Press, 1993.

- ▶ D. Welsh, Complexity: Knots, Colourings and Counting, London Mathematical Society Lecture Note Series, vol. 186, 1993.
- ▶ D. Lind, B. Marcus, An Introduction to Symbolic Dynamics and Coding, Cambridge University Press, 1995.
- ▶ J.H. van Lint, R.M. Wilson, A Course in Combinatorics, Cambridge University Press, 1996.
- ▶ R.L. Graham, M. Grötschel, L. Lovász, Handbook of Combinatorics, 2-volume set, The MIT Press, 1996.
- ▶ W.T. Tutte, Graph Theory As I Have Known It, Clarendon Press, 1998.
- ▶ B. Bollobás, Modern Graph Theory, Springer, 1998.
- ▶ A. Brandstädt, V.B. Le, J.P. Spinrad, Graph Classes: A Survey, 1999.
- ▶ C. Godsil, G. Royce, Algebraic Graph Theory, Springer, 2001.
- ▶ D.B. West, Introduction to Graph Theory, 2nd ed., Prentice Hall, 2001.

- ▶ B. Mohar, C. Thomassen, *Graphs on Surfaces*, Johns Hopkins University Press, 2001.
- ▶ S. Jukna, *Extremal Combinatorics, with Applications in Computer Science*, Springer, 2001.
- ▶ J. Matoušek, *Using the Borsuk-Ulam Theorem*, Springer, 2003.
- ▶ A. Schrijver, *Combinatorial Optimization (3 volume, A,B, & C)*, Springer, 2003.
- ▶ J.P. Spinrad, *Efficient Graph Representations*, AMS, 2003.
- ▶ M.C. Golumbic, *Algorithmic Graph Theory and Perfect Graphs*, 2nd ed., North-Holland, 2004.
- ▶ S.K. Lando, A.K. Zvonkin, *Graphs on Surfaces and Their Applications*, Springer, 2004.
- ▶ L.H. Harper, *Global Methods for Combinatorial Isoperimetric Problems*, Cambridge University Press, 2004.
- ▶ P. Hell, J. Nešetřil, *Graphs and Homomorphisms*, Oxford University Press, 2004.

- ▶ J. Kleinberg, E. Tardos, Algorithm Design, Addison Wesley, 2005.
- ▶ J.A. Bondy, U.S.R. Murty, Graph Theory, Springer, 2008.
- ▶ J. Bang-Jensen, G. Gutin, Digraphs: Theory, Algorithms and Applications, 2nd ed., Springer, 2008.
- ▶ W. Imrich, S. Klavžar, D.F. Rall, Topics in Graph Theory: Graphs and Their Cartesian Product, A K Peters, 2008.
- ▶ P. Flajolet, R. Sedgewick, Analytic Combinatorics, Cambridge University Press, 2009.
- ▶ M. Loeb, Discrete Mathematics in Statistical Physics, Vieweg+Teubner, 2010.
- ▶ R. Diestel, Graph Theory, Springer, 4th ed., 2010.
- ▶ J. Matoušek, Thirty-three Miniatures: Mathematical and Algorithmic Applications of Linear Algebra, AMS, 2010.
- ▶ A. Dress, K.T. Huber, J. Koolen, V. Moulton, A. Spillner, Basic Phylogenetic Combinatorics, Cambridge University Press, 2011.

None of the nice books listed above is now available at an affordable price in China mainland book market.

So, what will be provided to our course by the university textbook center is the following:

R. Balakrishnan, K. Ranganathan, A Textbook of Graph Theory, Springer, 2000.

Note that it is good to have such a textbook in hand but this does not mean that this course will follow that book.

Journals

Algorithmica

Algorithms

Annals of Combinatorics

Applicable Analysis and Discrete Mathematics

Applied Mathematics Letters

Ars Mathematica Contemporanea

Combinatorica

Combinatorics, Probability, and Computing

Computer Science Review

Discrete Applied Mathematics

Discrete and Computational Geometry

Discrete Mathematics

Discrete Mathematics & Theoretical Computer Science

Diskretnaya Matematika

Electronic Journal of Combinatorics

European Journal of Combinatorics

Geometriae Dedicata

Graphs and Combinatorics

Journals

Information Processing Letters

Integers: Electronic Journal of Combinatorial Number Theory

Journal of Algebraic Combinatorics

Journal of Algorithms

Journal of Combinatorial Theory, Series A

Journal of Combinatorial Theory, Series B

Journal of Complexity

Journal of Graph Theory

Journal of Graph Algorithms and Applications

Lecture Notes in Computer Science

Linear Algebra and its Applications

Moscow Journal of Combinatorics and Number Theory

Online Journal of Analytic Combinatorics

Order

Seminaire Lotharingien de Combinatoire

SIAM Journal on Computing

SIAM Journal on Discrete Mathematics

Theoretical Computer Science

<http://www.cs.technion.ac.il/~janos/COURSES/238901-07/>

<http://www.math.tau.ac.il/~rshamir/atga/atga.html>

<http://www.math.tau.ac.il/~nogaa/topics.html>

<http://users.encs.concordia.ca/~chvatal/691/>

<http://cgm.cs.mcgill.ca/~breed/SRL/home.html>

<http://www.math.mcgill.ca/~jakobson/courses/math707.html>

<http://www.scs.ryerson.ca/~mth607/>

<http://math.tut.fi/~ruohonen/graph.html>

<http://www.warwick.ac.uk/~masgax/MA241.htm>

<http://www.maths.qmul.ac.uk/~keevash/MTH711U/>

http://www.uea.ac.uk/menu/acad_depts/mth/math/syllabuses/0607/3

<http://www.maths.qmul.ac.uk/~whitty/MTH6105/>

<http://www.dpmms.cam.ac.uk/~dce27/>

<http://www2.mat.dtu.dk/education/01227/>

<http://www2.imm.dtu.dk/projects/graph/Program.html>

<http://www.cs.elte.hu/~karolyi/GT/index.html>

<http://bolyai.cs.elte.hu/~lovasz/homs2010.html>

<http://bolyai.cs.elte.hu/~lovasz/geomgraph09.html>

<http://cit.mak.ac.ug/~staff/jquinn/teaching/graphtheory/>

<http://www.fi.muni.cz/hlineny/Teaching/AGTS.html>

<http://www.fi.muni.cz/hlineny/Teaching/AGTT.html>

<http://www.ti.inf.ethz.ch/ew/courses/Top11/>

<http://www.ti.inf.ethz.ch/ew/courses/GT03/>

<http://www.ti.inf.ethz.ch/ew/courses/AMC08/#literature>

<http://homepages.cwi.nl/lex/>

<http://www.win.tue.nl/aeb/srgbk/>

<http://iasl.iis.sinica.edu.tw/hsu/teach/2010/index.html>

<http://www.math.ntu.edu.tw/gjchang/courses/2007-02-perfect-graph/2007-02-perfect-graph.htm>

<http://www.jaist.ac.jp/uehara/course/2007/i618/index.html>

<http://www.mathematik.uni-muenchen.de/>

[kang/graphentheorie.php](http://www.mpi-kang/graphentheorie.php)

<http://www.mpi->

[inf.mpg.de/departments/d1/teaching/ws11/ReadingGroup/ReadingGroup](http://www.mpi-inf.mpg.de/departments/d1/teaching/ws11/ReadingGroup/ReadingGroup)

<http://www.mpi->

[inf.mpg.de/departments/d1/teaching/ss11/TopologicalMethods/](http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/TopologicalMethods/)

<http://www.informatik.uni-rostock.de/ab/ps-files/basicalgor2.pdf>

<http://www2.cs.uni-paderborn.de/cs/ag-madh/vorl/GraphAlgorithmsI02/>
http://www.math.dartmouth.edu/archive/m68f07/public_html/
<http://www.math.princeton.edu/snorin/matchingMinicourse.html>
<http://www.math.cmu.edu/pikhurko/>
<http://www.ma.utexas.edu/users/geir/teaching/m390c/m390c.html>
<http://www.cs.yale.edu/homes/spielman/eigs/>
<http://math.stanford.edu/fthorne/math108/index.html>
<http://cseweb.ucsd.edu/classes/wi11/cse21/>
<http://www.math.ucsd.edu/fan/teach/262/07w.html>
<http://www.math.cornell.edu/levine/18.312/>
<http://ocw.mit.edu/courses/mathematics/18-315-combinatorial-theory-introduction-to-graph-theory-extremal-and-enumerative-combinatorics-spring-2005/lecture-notes/>
<http://www.math.lsa.umich.edu/fomin/665f09.html>
<http://www.cs.washington.edu/homes/jrl/tocmath08/>
<http://www.math.washington.edu/billey/classes/582.html>
<http://people.math.gatech.edu/thomas/TEACH/6014/>
<http://www.cs.bilkent.edu.tr/ugur/teaching/cs570/>
<http://vigna.dsi.unimi.it/fibrations/>

Course Description of MAT 218C: Topics in Discrete Mathematics

The aim of the course is to introduce students to Combinatorics, a fundamental mathematical discipline as well as an essential component of many mathematical areas. While in the past many of the basic combinatorial results were obtained by ingenuity and detailed reasoning, the modern theory has grown out of this early stage and often relies on deep, well-developed tools. The course will cover over a dozen virtually independent topics, chosen to illustrate several such techniques. This is an ultimate fun course, showcasing the gems of modern Combinatorics. – Benny Sudakov

An elegant proof is a proof which would not normally come to mind, like an elegant chess problem: the first move should be paradoxical. – Claude Berge

The mathematician's patterns, like the painter's or the poet's must be beautiful; the ideas, like the colors or the words must fit together in a harmonious way. Beauty is the first test: there is no permanent place in this world for ugly mathematics. – G.H. Hardy

Mathematics, rightly viewed, possesses not only truth, but supreme beauty – a beauty cold and austere, like that of sculpture. – Bertrand Russell

L. Lovasz, A perfect notion (to the memory of Claude Berge),
July 2004, ppt available at [http://bolyai.cs.elte.hu/
lovasz/presentations.html](http://bolyai.cs.elte.hu/lovasz/presentations.html)

Donald E. Knuth, The Sandwich Theorem, Electronic Journal of
Combinatorics, # A1, 1994.

M. Aigner, G.M. Ziegler, Proofs from the BOOK, Springer.

Plan of the Course (Tentative)

Graph connectivity, connected graphs (Knuth), 2-connected graphs, 3-connected graphs (Tutte)

List colouring of planar graphs (Thomassen), How to guard a museum (Chvatal), First-Fit colouring of interval graphs

From Shannon capacity to Perfect Graphs, various classes of perfect graphs, min-max theorems, minimally imperfect graphs, matching theory (Hall, Tutte, Gale-Sharpley)

Linear Programming and Transversal of d -interval

From Shannon capacity to Lovasz θ -function, Sperner capacity, geometric representation of graphs, dot-product dimension of graphs, quantum Lovasz θ -function

Tree-like structures and treelikeness parameters of graphs, Phylogenetic combinatorics.

Graph homomorphisms, Finite common coverings of graphs, Symbolic dynamics and coding, graph dynamics

Friendship theorem, directed Friendship theorems, Strongly regular graphs (digraphs)

circular-arc hypergraph, chordal graphs, graphical models

It's OK to be Stuck

It's OK to be stuck: As we study finite mathematics everyone will have his or her moments of confusion followed by periods of elation when it makes sense. How a student reacts to being confused affects his or her ultimate success. It is normal and natural to be stuck. Stopping in quick frustration or becoming panic stricken are not helpful responses but they are very common. Persistence is critical. Think calmly, look at previous problems, read the textbook, and ask for help. – Jim Davis, <http://www.indiana.edu/jfdavis/teaching/m118/M118syllabus.pdf>

2011年度本系组织的组合会议

- ▶ 2011.05.12 Minisymposium on Combinatorics and Data Science
- ▶ 2011.05.24-05.28 Seventh Shanghai Conference on Combinatorics
- ▶ 2011.09.15-09.19 Workshop on Algebraic Combinatorics at Shanghai Jiao Tong University

Nesetril的建议

李国伟：你有什么建议给想进入组合数学领域的学生？

Nesetril: 嗯, 我会告诉他们不要左顾右盼, **最好专心在研究上**, 坐下来, 看看他们可以做些什么。但良师益友, 倒是应该多方寻找结交。我认为一个人身边的朋友, 老师和同事都很重要, 因为**这是你个人的选择, 得看你有没有智慧, 懂不懂方法**。我的意思是, 这是很见仁见智的事。你遇到十个人, 然后从中挑一个作为指导教授。但当然, 学生得自己决定, 而且他们没办法得到任何建议, 要不然就是很少。不会有老师说某某人不好, 他们也不常说某人不错。所以学生们不止要能识人, 还要很幸运。我想这是每个人生涯上的一个关键转折, 所以, 你非得跟对的人一起共事才行。谁是你名义上的老师其实不重要, 总是会有人在文凭上签字。但是**你得有一群好友在身边, 好的同事与真正的良师, 这非常重要**。对我们老师来说也一样。我们得快速发掘人才, 知道谁表现杰出。

要做一个好的教师，很不容易，必需自己有相当的根底，才能讲好。应该做到以十当一，自己会十，但讲出来的是一，而现在有些教师是以一当十，这怎么教得好呢？—许宝騵

本课程不保证每个人都可以及格，不追求让每个人每次都能听懂，而且不属于数学系荣誉课程，不能给选修同学任何荣誉感，对就业，出国，升学，攒学分等等目的而言本课程未必能起到正面作用。授课教师对自己所授内容理解不深，需要现学现想现卖，时常汗流满面但仍然由于眼光浅薄而陷入迷途。选择的课程题材全凭一时兴趣，既不顾及应用，也不对大众口味，甚至不愿意追随任何教学大纲，也不赞成制定任何详细的教学大纲。

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