Algebraic Topology Final Exam 2009

1. Let E_1 and E_2 be two subspaces of a topological space E such that $Int(E_1) \cup Int(E_2) = E$. Show that $H_p(E, E_1 \cap E_2) = H_p(E_1, E_1 \cap E_2) \bigoplus H_p(E_2, E_1 \cap E_2)$.

2. Let S_r be an r-simplex and $\dot{S}_r$ its boundary. Show that $H_r(S_r, \dot{S}_r)$ has a generator represented by a relative cycle which is a linear map from the standard r-simplex onto S_r.

3. Let K be a chain complex such that all the K_p are free abelian groups and $H_p(K)$ vanishes for every p. Let f be a chain homomorphism from K to the zero chain complex. Show that f has a homotopy inverse.

4. Suppose that we have a chain homotopy $f \simeq f' : C \to C'$ and a chain homotopy $g \simeq g' : D \to D'$. Prove that there exists a chain homopoty $f \bigotimes g \simeq f' \bigotimes g' : C \bigotimes D \to C' \bigotimes D'$.

5. Compute the cohomology ring of the torus T^2.