1: 50 – 3:50 pm, January 8, 2008, Shang Yuan 102

Graph Theory Final Exam

Name: ____________ Student ID: ____________ Score: ____________

You should say what you mean and mean what you say!

1. (15 marks) Let G be an irreducible digraph with spectral radius 2008. Show that we can use a series of out-state splitting to transform G into a digraph with constant out-degree 2008.

2. (15 marks) Let G be a finite digraph and (X_G, σ) the edge shift associated with G. Show that the zeta function of (X_G, σ) is $\zeta_{X_G}(t) = \prod_C (1-t^{|C|})^{-1/|C|}$, where C runs through all closed walks of G and $|C|$ denotes the length of C.

3. (10 marks) Suppose that A, B are two complex square matrices, f is a polynomial, μ is an eigenvalue of A and $f(\mu) \neq 0$. Prove the following: If there are matrices R and S such that $BS = SA$ and $RS = f(A)$, then μ is an eigenvalue of B.

4. (15 marks) Let A be an $n \times n$ integral matrix and $p(t) \in \mathbb{Z}(t)$ such that $p(0) = \pm 1$. Define the generalized Bowen-Franks group $BF_p(A)$ to be $\mathbb{Z}^n/\mathbb{Z}^n p(A)$. Show that $BF_p(A)$ is an invariant of shift equivalence over $\mathbb{Z}$.

5. (15 marks) An integer matrix is shift equivalent to $[n]$ over $\mathbb{Z}$ if and only if its zeta function is $\frac{1}{1-nt}$.

6. (20 marks) Let A and B be integral matrices. Suppose that $(\Delta_A, \delta_A) \simeq (\Delta_B, \delta_B)$. Show that $A \sim_Z B$.

7. (10 marks) Determine the dimension group of $A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$.