Review: [Untitled]

Reviewed Work(s):

An Introduction to Symbolic Dynamics and Coding. by Douglas Lind; Brian Marcus

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approximation theory. It will be very useful for students.

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An Introduction to Symbolic Dynamics and Coding. *By Douglas Lind and Brian Marcus.* Cambridge University Press, Cambridge, UK, 1995. $27.95. xvi + 495 pp., softcover. ISBN 0-521-55900-6.

The title of this fine book reflects its scope.

Section 1. *Symbolic Dynamics.* Symbolic dynamics first emerged as a tool in the study of dynamical systems, especially geodesic flows. Here is the initial rough idea. Suppose we have n disjoint sets (named $1, 2, \ldots, n$) in a space through which a point x moves under some action of $\mathbb{N}$, $\mathbb{R}$, or $\mathbb{Z}$. Associate with x a sequence y, where (say) x is initially in the set y_0 and successively visits $y_1, y_2, y_3, \ldots$. In some situations, one can deduce the existence or preponderance of interesting phenomena by sufficiently constraining the set of sequences which arise from the given action.

To go beyond sets, one makes a topological dynamical system out of the symbolic dynamics. Begin with the set $X_{[n]} = \{1, \ldots, n\}^{\mathbb{Z}}$ of all doubly infinite sequences on n letters $1, \ldots, n$. With the natural topology, $X_{[n]}$ becomes a compact metrizable space with $\text{dist}(y, y') = 1/(k+1)$ if k is the smallest nonnegative integer such that $y_i = y'_i$ whenever $|i| < k$. The shift homeomorphism $\sigma \colon X_{[n]} \to X_{[n]}$ is defined by $(\sigma x)_i = x_{i+1}$. The pair $(X_{[n]}, \sigma)$ is the topological dynamical system known as the full shift on n symbols. The restriction of this shift to a closed shift-invariant subset is a subshift.

Arguably the most important class of subshifts are the shifts of finite type (SFTs). Every SFT is topologically conjugate to an "edge" SFT σ_A, presented by some square nonnegative matrix A. Here A is viewed as the adjacency matrix of a directed graph, whose edge set is the alphabet of the associated SFT. The domain X_A of σ_A is the space of doubly infinite sequences x corresponding to walks through the graph; i.e., for every i, the terminal vertex of x_i equals the initial vertex of x_{i+1}. The SFTs are important tools in hyperbolic dynamics [Bow], are fundamental units for constructions in ergodic theory and topological dynamics [DGS], and provide the natural setting for applications of symbolic dynamics to matrices [Boy] and coding.

Section 2. *Coding.* In computer science and engineering it is useful to consider finite automata which produce output tapes from input tapes by the repeated application of a finite local rule (sliding block code). Then it is very natural to idealize the possible input and output tapes as spaces of infinite sequences and consider codes which map one space into another. For example, one might face the problem of efficiently storing arbitrary data into some magnetic medium, and this may translate into the problem of efficiently coding arbitary sequences of zeros and ones into a space of sequences subject to constraints imposed by the storage device. Problems of this sort have found a natural mathematical framework in symbolic dynamics. In particular, homomorphisms between shifts of finite type (as topological dynamical systems) are exactly the maps induced by sliding block codes. Some of the ideas already developed for studying topological relations among SFTs, with no hint of application, might have been tailor-made for coding problems. This connection was first pointed out explicitly in [ACH], which won an annual IEEE Information Theory Group Award for the best paper on information theory. The conceptual parallels among symbolic dynamics, coding theory, automata theory, and system theory are spelled out in [M] and [FMST].

The book under review covers very little of traditional error correction coding as in, say, the introduction [V]. The book is concerned with symbolic dynamics with an emphasis on its pertinence to coding problems.

Section 3. *The Introduction.* The book carefully builds the fundamental framework for the study of shifts of finite type, sofic systems, and their coding relations and internal structure. The topics include entropy, periodic points and zeta functions, shift equivalence, strong shift equivalence, finite to one codes, lower entropy factors, Krieger's embedding theorem, existence of nonnegative matrices with prescribed algebraic invariants, markers, degrees of codes, almost topological conjugacy, canonical covers of sofic systems, resolving and closing maps, the dimension group, and ideal classes. Connections to data storage and transmission are covered. The book is self contained; for example, the required results from the Perron–Frobenius theory of nonnegative matrices are proved.

The book is a model of organization and clarity, consistent in pace and tone, with very few errata. There are plenty of useful exercises, though not many very hard ones. The book aims to be accessible to engineers and even strong undergraduates, and the mathematical prerequisites are correspondingly modest (for example, measure theory is not required). However, it still works well for readers with stronger backgrounds. I was pleased with it as the main source for a one-semester graduate course, supplemented with more advanced material.

The book is even better for individual reading. A graduate student can move steadily through it with very little help. The number of pages (approaching 500) is deceptive: clarity supersedes conciseness and very little proof is left to the reader. The index and index of notation are excellent.

The book is emphatically an introduction. The authors omit some important difficult proofs (e.g., Ashley's theorem on the existence of closing maps) and do not develop various central or important topics (e.g., the thermodynamic formalism, Markov measures, automorphisms of the shift, countable-state Markov chains). On the other hand, with comments and examples throughout the book and a 40 page survey of advanced topics at the end, the authors give quite a good (if sometimes cursory) overview of the open problems and research activity in the field today and provide an excellent sense of its historical development. The survey comes with references to a comprehensive bibliography. Future monographs will be easier with this book as a basic reference.

In summary, the book is a very fine and broadly accessible introduction to symbolic dynamics and coding. It is now an essential reference which fills a gaping hole in the literature.

Section 4. *The Website.* The website

http : //www.math.washington.edu/SymbolicDynamics

contains a detailed table of contents of the book and a copy of its preface. The preface includes a fine sketch of the genesis of symbolic dynamics and a useful discussion of the organization and contents of the book.

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