⟨⟨ 拓扑学基础 ⟩⟩ 期中考试

姓名：　　　　　　　学号：　　　　　　　成绩：

一 a 、 (15 分) Prove that every second countable space is seperable 。

一 b 、 (15 分) Prove that any continuous real valued function f on S^1 must identify a pair of antipodal points.

二 a 、 (20 分) For any $A \subseteq R^2$, define its interior to be $\{y \in A : \forall h \in R^2, \exists t_h > 0,$ such that $y + th \in A, \forall t \in [0, t_h]\}$. Show that this interior operator satisfies the axiom for interiors and so does define a topology on R^2, which we denote by $\mathcal{O}$ 。 Associate with $R^2 \times R^2$ the product topology $\mathcal{O}'$ arising from $\mathcal{O}$. Show that the map f from $(R^2 \times R^2, \mathcal{O}')$ to $(R^2, \mathcal{O})$ given by $f(x, y) = x + y$ is not continuous.

二 b 、 (20 分) Prove that $GL(n, \mathcal{C})$ is path connected.

二 c 、 (20 分) The map $\alpha : R^{1+n} \to \mathcal{C}(R, R)$ assigns to each $(n+1)$-vector $(a_0 a_1 \dots a_n)$ the polynomial $a_0 + a_1 t + \cdots + a_n t^n$ in the real variable t. Show that α is continuous 。 (Hint: Use the Theorem of Exponential Correspondence)

三 a 、 (15 分) Show that there exists a bounded complete metric space which is not compact.

三 b 、 (15 分) Prove that continuous maps preserve convergent sequences. Explain why the product topology is called the topology of pointwise convergence.

四 a 、 (20 分) Let X be a T_4 space and $U_1, \dots, U_k$ an open covering of it. Show that we can find another open covering of X, say $V_1, \dots, V_k$, such that $V_i \subseteq \overline{V_i} \subseteq U_i, \forall i$.

四 b 、 (20 分) Consider the product space $X \times Y$, where Y is compact. Let N be an open set of $X \times Y$ containing the slice $x_0 \times Y$ of $X \times Y$. Show that N contains some tube $W \times Y$, where W is a neighborhood of x_0 in X.

五 a 、 (20 分) Give the definition of a Lebesgue number for an open covering of a metric space. Prove that any continuous map from a compact metric space to a metric space is uniformly continuous.

五 b 、 (20 分) Give the definition of a homotopy between two continuous maps. Explain clearly the relationship between homotopy and the path connected components of the mapping space with the compact open topology.

六、 (30 分) Let X, Y be two topological spaces. For any map $f : X \to Y$, set $Gr(f) = \{(x, f(x));\ x \in X\}$ 。 Prove the following: 1. f is continuous if and only if the map g from X to $X \times Y$ which sends x to $(x, f(x))$ is continuous; 2 If f is continuous and X is path connected, then $Gr(f)$ is path connected; 3 If f is continuous and Y is Hausdorff, then $Gr(f)$ is closed in $X \times Y$.