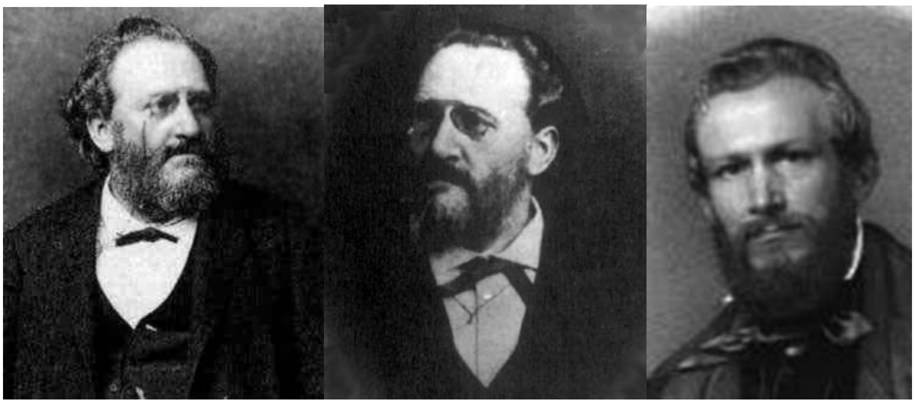
Paul David Gustav du Bois-Reymond



Born: 2 Dec 1831 in Berlin, Germany
Died: 7 April 1889 in Freiburg, Germany

Paul du Bois-Reymond's parents were Felix Henri du Bois-Reymond and Minette Henry. Felix du Bois-Reymond was from Neuchâtel but moved to Berlin in 1804 where he was a teacher at the Kadettenhaus. Switzerland had been conquered by Napoleon in 1798 who then established the Helvetic Republic, which lasted until 1803. France imposed a constitution which had no respect for Swiss traditions and there was much internal disorder. Napoleon intervened in 1803 with the Act of Mediation which substituted a new Swiss Confederation for the Helvetic Republic, producing even closer ties with France. Neuchâtel, however, was not part of the Swiss Confederation at this time and was under the nominal control of the king of Prussia. With the move to Berlin, Felix remained in Prussia and he maintained a close association with Neuchâtel. However, he had great affinity to France and his wife Minette was daughter of the minister for the French colony in Berlin. Later Felix acted as a representative from Neuchâtel to the Prussian government.

Felix and Minette du Bois-Reymond had five children, Emil born in 1818, two daughters Julie and Felice, and two further sons one of whom was Paul, the subject of this biography. Felix was a Pietist, a religious movement which sprung up in Germany and spread beyond that country. Pietism, a branch of Protestant Christianity, emphasized involvement of people in religion, piety and learning. The du Bois-Reymond family were brought up very strictly with Felix exerting strong authority over his children. Although living in Berlin, they spoke French at home and their eldest son Emil attended the French Gymnasium in Berlin, spending a year studying in Neuchâtel. It was an upbringing which saw Paul totally fluent in both French and German. There was 13 years difference in age between Paul and the older Emil, and Paul was strongly influenced by Emil who went to the University of Berlin when Paul was only a six year old child.

Like his older brother, Paul also attended the French Gymnasium in Berlin and he continued to follow in his brother's footsteps by attending the Collège in Neuchâtel. By this time Emil had become a famous physiologist and Paul decided that he would follow his older brother into the same career. From Neuchâtel, Paul went to the Gymnasium in Naumburg and then entered the University of Zurich in 1853. Emil had been elected to the Prussian Academy of Science two years earlier and Paul, striving to follow, began to study medicine. In 1854, Paul du Bois-Reymond published four papers which studied physiological problems. Moving to Königsberg he was influenced by Franz Neumann to change to mathematical physics. Still at this stage he was part scientist making experimental observations, part theoretical mathematician trying to fit his observations of liquids into a mathematical theory. His doctoral studies were supervised by Kummer and du Bois-Reymond was awarded his doctorate by the University of Berlin in 1853 for his thesis *De aequilibrio fluidorum.*

After his doctorate du Bois-Reymond was appointed to teach mathematics and physics at a secondary school in Berlin. However, he continued to undertake research into applied mathematics and, as a consequence, became more and more involved with the theory of partial differential equations. In 1864, while still teaching at the secondary school, du Bois-Reymond published *Beiträge zur Interpretation der partiellen Differentialgleichungen mit drei Variabeln.* In this work he generalized Monge's idea of the characteristic of a partial differential equation from second order equations to third order equations. This work formed a basis of what Lie was to generalize later. Following the publication of this important work, du Bois-Reymond was appointed to a chair at the University of Heidelberg in 1865. After five years in Heidelberg he moved to a chair at the University of Freiburg where he taught from 1870 until 1874 when he was appointed to the chair at the University of Tübingen where he succeeded Hankel. His period at Freiburg was made rather more difficult by the Franco-Prussian war which saw France rapidly defeated by Prussia in the war of 1870-71. Du Bois-Reymond's strong French and Prussian links put him in a somewhat difficult position and his brother Emil was an outspoken critic of the French in this dispute. Finally, after ten years at

Tübingen, where he supervised the doctorates of a number of students the most famous of whom was Otto Hölder, du Bois-Reymond was appointed to a chair at the Technische Hochschule Charlottenberg in Berlin. Although du Bois-Reymond got on well with Weierstrass and the two shared many similar mathematical interests and concerns for rigour, the same could not be said for members of Weierstrass's school with whom relations were strained. In particular du Bois-Reymond and Schwarz were not on good terms.

Du Bois-Reymond's work is almost exclusively on calculus, in particular partial differential equations and functions of a real variable. The standard technique to solve partial differential equations used Fourier series but Cauchy, Abel and Dirichlet had all pointed out problems associated with the convergence of the Fourier series of an arbitrary function. In 1873 du Bois-Reymond was the first person to give an example of a continuous function whose Fourier series diverges at a point. Perhaps what was even more surprising, the Fourier series of du Bois-Reymond function diverged at a dense set of points. The important work *Eine neue Theorie der Convergenz und Divergenz von Reihen mit positiven Gliedern* ("A new theory of convergence and divergence of series with positive terms") led to an increasing understanding of the whole concept of a function.

Du Bois-Reymond published an example of a continuous function which is nowhere differentiable in 1875. It was inspired by a similar function found by Weierstrass in 1872 but not published by him until much later. This example contradicted most mathematicians' intuition, for it was generally believed that a continuous function was differentiable everywhere except in special points. Du Bois-Reymond wrote:-

It appears to me that the metaphysics of Weierstrass's function still hides many riddles and I cannot help thinking that entering deeper into the matter will finally lead us to the limits of our intellect.

Although there is no clear evidence that Cantor was guided to his "diagonal argument" from du Bois-Reymond's work, there is clear evidence that du Bois-Reymond had essentially found the diagonal argument in 1875. Although Cantor proved that the real numbers are uncountable one year earlier he did not find the much clearer diagonal argument until some years later.

In 1880 du Bois-Reymond pointed out the importance of nowhere dense sets which cannot be appropriately covered. Cantor wrote in 1883:-

In the investigation of du Bois-Reymond ... on generalizations of theorems on integration those point sets are used which have the property that they can be covered by a finite number of intervals so that the sum of all intervals is smaller than an arbitrarily given quantity.

Du Bois-Reymond published *Die allgemeine Functionentheorie* in 1882.

It is a remarkable work in many ways, although its significance was not fully appreciated at the time. In it he claimed that there are many important mathematical results which will never be proved either true or false, but did not attempt to put this assertion into a formal setting. Also in this book he discussed the real numbers, the continuum, and space:-

The conception of space as static and unchanging can never generate the notion of a sharply defined, uniform line from a series of points however dense, for, after all, points are devoid of size, and hence no matter how dense a series of points may be, it can never become an interval, which must always be regarded as the sum of intervals between points.

He believed that a full understanding of the continuum was beyond the capabilities of mathematicians. However he had already developed a theory of infinitesimals in *Über die Paradoxen des Infinitär-Calcüls* ("On the paradoxes of the infinitary calculus") in 1877. He writes:-

The infinitely small is a mathematical quantity and has all its properties in common with the finite ... A belief in the infinitely small does not triumph easily. Yet when one thinks boldly and freely, the initial distrust will soon mellow into a pleasant certainty ... A majority of educated people will admit an infinite in space and time, and not just an "unboundedly large". But they will only with difficulty believe in the infinitely small, despite the fact that the infinitely small has the same right to existence as the infinitely large ...

Were the sight of the starry sky lacking to mankind; had the race arisen and developed as cave dwellers in enclosed spaces; had its scholars instead of wandering through the distant places of the universe telescopically, only looked for the smallest constituents of form and so were used in their thoughts to advancing into the boundless in the direction of the immeasurably small: who would doubt then that the infinitely small would take the same place in our system of concepts that the infinitely large does now? Moreover, hasn't the attempt in mechanics to go back down to the smallest active elements long ago introduced into science the atom, the embodiment of the infinitely small? And don't as always skilful attempts to make it superfluous for physics face with certainty the same fate as Lagrange's battle against the differential?

Although never a mathematician of the first rank, nevertheless the contributions by du Bois-Reymond during the 1870s and early 1880s were highly significant. Let us list some of the 1870s papers: *Notiz über einen Cauchy'schen Satz, die Stetigkeit von Summen unendlicher Reihen betreffend* (1871); *Die Theorie der Fonrier'schen Integrale und Formeln* (1871); *Über asymptotische Werthe, infinitäre Approximationen und infinitäre Auflösung von Gleichungen* (1875); *Zusätze zur Abhandlung: Untersuchungen über die Convergenz und Divergenz der Fourier'schen Darstellungsformeln* (1876); *Notiz über infinitäre Gleichheiten* (1876); *Zwei Sätze über Grenzwerthe von Functionen zweier Veränderlichen* (1877); *Note über die Integration totaler Differentialgleichungen* (1877); *Notiz über Convergenz von Integralen mit nicht verschwindendem Argument* (1878); and *Über Integration und Differentiation infinitärer Relationen* (1879).

As Novy writes in [1]:-

Du Bois-Reymond's work was directed at the basic questions of mathematical analysis of the time and is marked by both the personality of the author and the state of mathematics of the period.

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