Analysis of seat availability of library

Xiao Tao  Chen Weichong  Li Jiannan  Chen Huaiyi  Zhu Enze

Abstract: With the advent of the final exam, the utilization ratio of the seats in the libraries are more and more cared by the librarians and students. Based on the analysis of the seats utilization ratio of Bao Yugang Library in Shanghai Jiaotong University, the essay will reveal its inherent laws. Because the students always have self-studing from 8 o'clock in the morning to 11 o'clock. During this period, the occupation of seat will bring trouble to the students who don't have a seat yet. In this essay we will pay most attention to total number of the students in library during the period from 8 o'clock in the morning to 11 o'clock. And then we can get the inherent laws. And for the utilization ratio during 11 o'clock to 22 o'clock we can use the same way to have a deep research so we will not say too much about that. When discussing the total number of the students in the library and the equation of the seat occupation during 8 o'clock to 11 o'clock, with the fitting of the data, we can divide the function into three sections: 8:00 to 8:40, 8:40 to 9:30 and 9:30 to 11:00, and gives a specific function expressions and images. The readers can clearly see the total number of students seats occupation in any time point. Then they can determine the best time to go to the library so that they can find a seat. At the end, we have some suggestions for the librarians and the students.

Keyword: differential equation model, seat availability, exponential growth, inflow of students, total number of students, number of seat occupation

1. Research background

With the increasing world population, people can use less and less resources, and the waste of resources has also become a big problem. Under the population pressure the traffic became heavy and space resources is also under tense situation. And the greater the population, will also result in a corresponding decline in energy efficiency. A lot of resources for group activities is of a serious waste, especially in public places, the waste is particularly serious. To make full use of resources, to reduce waste, we must try in any aspects to improve energy efficiency,

There's only ordinary seats in the libraries, although the library has clearly defined
that students can not occupy seat. But because of the lack of the alarm system, we cannot judge which one is free and which one is not, so the problem of the low utilization ratio of seats is very serious. This leads to a serious waste of resource and so many students who loves to study can not find seats in the library, besides the students who have seat can not study with peace in mind. School rules have been gradually forgotten by some students, this has also brought a lot of trouble to the management of the librarians. So that is a problem which should soon be solved.

2. Question assumptions and symbols explanations

2.1 Question assumptions

(1) Since most students enter the library to study, we assume that each student can find an appropriate seat.

(2) Students may leave the seat for a short time for some special reasons such as telephoning or going to the washroom. Then we still think that the seats are being used.

(3) Because the same person will not enter and leave the library too often from 8:00 to 11:00, we take the person-time as the number of the total number of the students.

(4) Total number of students within the library and the number of seat occupation are not just a function of time, but also effected by many other factors, such as the temperature outdoor, the holidays and weather the air-conditioned classrooms are opened. But this article focuses more on revealing the inherent laws of its variation with time every day, so this essay will not take these factors into account.

2.2 Symbols explanations

Number of seat occupation: \( h(t) \)

Total number of students in the library: \( x(t) \)

Increasing ratio of the number of seat occupation: \( r(x) \)

Maximum capacity of the library: \( x_m \)

Utilization ratio of seats: \( \omega \)

Ratio of the seat occupation number and capacity: \( \varphi \)

3. Modeling

The function is based on the actual situation to express the number of the seats
occupied at \( t \).

The increment of the number of seats occupied in a library is proportional to the number of the students in it. Then the increment \( r \) of the number of the seats occupied \( h(t) \) is the function with respect to the number of students in the library \( x \), i.e.

\[
\frac{dh}{dt} = r(x)x \tag{1}
\]

Because the number of seats occupied in the library is limited by the capacity of the library, the growth cannot be unlimited. Thus, with the growth of the number of all students in the library, the growth rate will be decreasing linearly, i.e.

\[
r(x) = r - sx, \quad r, s > 0
\]

When the number of the students in the library reach the maximum capacity \( x_m \), i.e., \( x = x_m \), the growth rate should be equal to zero, i.e. \( r(x_m) = 0 \), from which we have: \( s = \frac{r}{x_m} \). Put it into (2), we have:

\[
r(x) = r \left(1 - \frac{x}{x_m}\right) \tag{3}
\]

Then substitute (3) into (1), we get:

\[
\frac{dh}{dt} = r \left(1 - \frac{x}{x_m}\right)x \tag{4}
\]

According to the actual situation, we find that there presents a certain law between the number of the students in the library \( x \) and time \( t \). Because no completely continuous function can be researched in real life, we come up with discrete model.

First, we assume that the change of \( x \) is an intrinsic function, i.e.

\[
\frac{dx}{dt} = f(t) \tag{5}
\]

Rewrite the differential form of (5), we get:

\[
\frac{x(t_k + \Delta t) - x(t_k)}{\Delta t} = f(t_k), k = 0, 1, 2, ...
\]

To simplify the statistics and to get expression of \( f(t) \), we use discrete time to collect the flow of students in and out of Bao Yugang Library in SJTU from December the 16th to January 6th, during which January 1st is a National Holiday without data to be taken into account.

Because the frequency of the students entering and leaving the library is relatively
high, we let $\Delta t=10\text{min}$, i.e. $\Delta t=0.167\text{h}$ in Equation.(6)

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Table1. *the change of the number of the students in the Bao Yugang Library from 8:00 to 9:00 everyday from December 26th to January 6th.*

Multiply each datum in December 26th by 6 and then fit them into a curve, we get

![Figure1](attachment:image.png)

*Figure1. the change of the number of the students in the Bao Yugang Library from 8:00 to 9:00 everyday from December 26th to January 6th.*

From the curve above we get that the net inflows of students of Bao Yugang Library shows a exponential growth trend. Because the growth of data after 8:40 am
destruction smoothness, so we cannot consider the two datum of 8:40 to 8:50 and 8:50 to 9:00. That is, total growth of students from 8:00 to 8:40 in the library is an exponential function. By fitting the data, we can get equation:

$$\frac{dx}{dt} = 357.3e^{2.8t}$$  \hspace{1cm} (7)

Combine differential equation (7) with the initial conditions, solution is:

$$x(t) = 127.6e^{2.8t} - 127.6, (0 \leq t \leq 0.667)$$  \hspace{1cm} (8)

Thus, we get the function of the number with respect to the time from 8:00 am to 8:40 am on December 26th in the library, i.e. equation (8). We substitute data from December 27th to January 6th into equation (8). In addition to the data of January 2 and January 3 whose difference between each other is relatively large. The remaining dates of the data mostly meet the equation (8). Since the temperature on January 2 and January 3 is relatively low, and those are also in a holiday, most students choose to review the lessons in the library, so in these two days, the number of the students in the library grew rapidly, which is of large difference of the equation (8). However, we can still approximate the function (8) as a function of the number of the students with respect to time in Bao Yugang Library from 8:00 am 8:40. The figure of equation (8) is as below:

Figure 2. number of students in the library from 8:00 am to 8:40 in Bao Yugang Library

Since the net inflows of students of the Bao Yugang Library no longer meet the exponential relationship after 8:40 am, and in order to further reveal the inherent law to find \( f(t) \), data from 8:40 ~ 9:30 of the library will be fit as a net inflow of students.
December 26th  |  78  |  42  |  9  
December 27th  |  73  |  44  |  16 
December 28th  |  75  |  39  |  10 
December 29th  |  77  |  48  |  11 
December 30th  |  76  |  40  |  8  
December 31st  |  76  |  41  |  9  
January 2nd   |  78  |  41  |  10 
January 3rd   |  87  |  49  |  20 
January 4th   |  75  |  45  |  13 
January 5th   |  69  |  53  |  16 
January 6th   |  82  |  46  |  21 

Table 2. changes in students from Dec. 26 to Jan. 6 9:00 am to 9:30 in Bao Yugang Library(Unit: person)

From the data above on December 26 multiplied by 6 and data from 8:40 to 9:00 with the data fitting, we can get the following curve:

![Graph showing the changes in students from December 26th to January 6th from 8:40 am to 9:30 in Bao Yugang Library.](image)

Figure 3. changes in the number of students on December 26th from 8:40 am to 9:30 in library

Shown in Figure 3, we can see the net inflow of students in Pao Yue Kong Library from 8:40 am to 9:30 shows a downward trend line. As is easy to see, at this time \( f(t) \) is a straight line with negative slope. By fitting the data, we can get equation:

\[
\frac{dx}{dt} = -1364t + 1890
\]  

(9)

Since the function image is continuous, we can get \( x(0.667) = 697 \)

\[
x(t) = -682t^2 + 1890t - 261, (0.667 \leq t \leq 1.5)
\]  

(10)
Thus, we get the function of the number with respect to time on the December 26\textsuperscript{th} from 8:40 am to 9:30 in the library, equation (10). Substituting the data of December 27 ~ January 6 into equation (10), they are basically satisfied. We can approximate the function (10) as a function of the number with respect to time in Pao Yue Kong Library from 8:40 am to 9:30. Image of function (10) is as below:

![Image of function (10)](image)

**Figure 4. the total number of students in Library from 8:40 PM-9:30PM**

From 9:30 ~ 11:00 in the morning, at the entrance of the library inside and outside are in basic balance, we make intervals in equation (6) as a half-hour, i.e. $\triangle t = 0.5h$ to collect data, the net inflow of students is almost zero. Total number of students at this time in the library is in equilibrium, that is, the image is a straight line parallel to the t axis, whose slope is close to 0. This shows that the total number of students in library stabilized at around 1040.

In summary, we can get the piecewise function of the total number of students in the Bao Yugang Library from 9:30 to 11:00, and its function image:

\[
x(t) = \begin{cases} 
127.6e^{2.8t} - 127.6 & 0 \leq t < 0.667 \\
-682t^2 + 1890t - 261 & 0.667 \leq t < 1.5 \\
1039.5 & 1.5 \leq t < 3 
\end{cases}
\]  \hspace{1cm} (11)
We define the libraries for their utilization rate \( \omega(t) = \frac{x}{x_m} \times 100\% \). Then when the total number of students in the library is in stability of 1040, the calculated seat occupied rate is \( \omega = 86.67\% \). Visible, utilization rate of seats in library is high, showing that the students are able to take full advantage of libraries for resources. However, due to the total large number of students in the university and limited resources of seats in the library, there are many students still complaining about the phenomenon of occupying a seat.

The expression \( x(t) \) obtained, we can use equation (4) to solve the number of seats occupied \( h(t) \) in Bao Yugang Library from 8:00 ~ 11:00 in the morning. By the equation (4), on both sides of integral, we can get:

\[
h(t) = r \int x(t)dt - \frac{1}{x_m} \int x^2(t)dt
\]  
  
(12)

Figure 5. number of students in the library from 8:00 am to 11:00 a.m. in library.

Because there are about 1,200 seats in Bao Yugang Library, i.e. \( x_m = 1200 \). And equation (12), together with the one after integral include two unknown variables in total. We need only to collect two data from 8:00 to 8:40, 8:40 to 9:30 and 9:30 am to 11:00, the expression \( h(t) \) can be obtained. This help to simplified our data collection also, because the collection of number of seats is relatively difficult. It is difficult to capture the instantaneous value and more variables we collect, more imprecise the result will be. If we get its function with only two values its accuracy will be improved. Here we use the December 26 to calculate the value \( h(t) \) and use the other date \( h(t) \) to prove the
accuracy of the value of the expression obtained.

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<tr>
<th>Date</th>
<th>8:20</th>
<th>8:40</th>
<th>9:20</th>
<th>10:00</th>
<th>10:30</th>
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<tbody>
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<td>180</td>
<td>87</td>
<td>51</td>
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<td>December 27(^{th})</td>
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<td>90</td>
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<td>50</td>
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<td>December 28(^{th})</td>
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<td>88</td>
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<td>December 29(^{th})</td>
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<td>182</td>
<td>91</td>
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<td>49</td>
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<tr>
<td>December 30(^{th})</td>
<td>102</td>
<td>177</td>
<td>86</td>
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<td>49</td>
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<td>December 31(^{st})</td>
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<td>184</td>
<td>86</td>
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<td>January 2(^{nd})</td>
<td>156</td>
<td>167</td>
<td>73</td>
<td>42</td>
<td>38</td>
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<td>January 3(^{rd})</td>
<td>154</td>
<td>176</td>
<td>69</td>
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<td>January 4(^{th})</td>
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<td>January 5(^{th})</td>
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<td>January 6(^{th})</td>
<td>103</td>
<td>188</td>
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Table 3. *the distribution of the number of the seats occupied in Bao Yugang Library from 8:00 am to 11:00.*

Put equation (8) into equation (12), we have

\[
h(t) = 2.42e^{5.6t} + (45.57r - 9.69)e^{2.8t} + (13.57 - 127.6r)t + c
\]

measurement conditions given by the December 26, we have \( h(\frac{1}{3}) = 107 \), \( h(\frac{2}{3}) = 180 \),

substituting them into equation (13), the solution was: \( r = 0.154 \), \( c = 100.25 \).

Substituting r and c of the values into equation (13), the expression can be obtained, i.e.

\[
h(t) = 2.42e^{5.6t} - 2.67e^{2.8t} - 6.08t + 100.25 \quad 0 \leq t \leq 0.667
\]

From Table 3, December 27\(^{th}\) ~ January 6\(^{th}\), without January 2 and January 3, the data are basically consistent with equation (14). Since January 2 and January 3 was in a statutory holiday and the temperature is low, then the number of seats occupied deviate from the general situation. Therefore, we can look equation (14) as the number of seats occupied from 8:00 am to 8:40 in Bao Yugang Library. The image is:
Figure 6. from 8:00 am to 8:40, the total number of seats occupied in Bao Yugang Library.

Substitute (9) into (12), we have
\[ h(t) = -77.52t^3 + 537.08t^4 - (2273.3r + 10911.4)t^3 + (945r + 411.08)t^2 - (266r + 56.77)t + c \] (15)

measurement conditions given by the December 26, we have, \( h\left(\frac{2}{3}\right) = 180 \), \( h\left(\frac{4}{3}\right) = 87 \), and put them in (15), we have: \( r = 0.63 \), \( c = 150.05 \)

Put them in (15), we get \( h(t) \):
\[ h(t) = -77.52t^3 + 537.08t^4 - 1234.36t^3 + 1006.43t^2 - 221.2t + 150.05 \quad 0.667 \leq t \leq 1.5 \]

From Table 3, 12 27 ~ January 6 data basically meet the above function, the image is:

When \( t > 1.5 \), \( x(t) = 1039.5 \), put them in (12):
\[ h(t) = (1039.5r - 900.47)t + c \] (16)
Substitute \( h(2) = 51 \) and \( h(2.5) = 47 \) into (16), we have:

\[
r = 0.86 \quad , \quad c = 64
\]

And put them in (16), we get the expression of \( h(t) \):

\[
h(t) = -6.5t + 64
\]  \hspace{1cm} (17)

From Table 3, data of December 26\(^{th}\) ~ January 6 basically meet the above function, the image is:

![Graph](image)

Figure 8. 9:30 am ~ 11:00, total number of seats occupied in Bao Yugang Library

We can see from the images, during 9:30 to 11:00 in the morning the number of seats occupied is not vibrating greatly, so we can take \( h(t) \) as a stable state, and its value fluctuated around 50.

In summary, we can get the piecewise function of seats occupied in the morning 9:30 am to 11:00 am in library, and its function image:

\[
h(t) = \begin{cases} 
2.42e^{5.6t} - 2.67e^{2.8t} - 6.08t + 100.25 & 0 \leq t < 0.667 \\
-77.25t^5 + 537.08t^4 - 1234.36t^3 + 1006.43t^2 - 221.2t + 150.05 & 0.667 \leq t < 1.5 \\
-6.5t + 64 & 1.5 \leq t < 3
\end{cases}
\]
Figure 9 shows the stable number of seats occupied 50 seats only occupied

\[ \varphi = \frac{50}{1200} \times 100\% = 4.17\% \]

which also shows from the other side that the seats utilized is relatively high in Bao Yugang Library.

Some reasonable suggestions:

1. Librarians should pay more attentions and take strict and effective measures managing the seats when the phenomenon of occupying seats reach its climax, i.e. about 8:30. Clear away the books and paper using for occupying seats, make room for the newcomers.

2. Considering the lack of librarians and massive workload, maybe it's a wise choice to develop a timing system, if the seat isn't used for 30 mins, the system will mark it available. Students can search which seat is available by the system.

3. To improve the utilization ratio of public source, students can leave a note, saying what time he leaves and comes back, or in which time interval this seat is available. Other students can use this seat during the available time.

4. To reduce workload, administrative department can install more camera in the library. Librarians can keep a watchful eye on study area, clearing away the books using for occupying seats.

5. Open more air-conditioned classroom to satisfy students' needs, especially in summer and winter. We think this is the most effective way.