Randomized $\Delta$-Edge-Coloring via Quaternion of Complex Colors

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Abstract

This talk explores the application of a new algebraic method of color exchanges to the edge coloring of simple graphs. Vizing’s theorem states that the edge coloring of a simple graph $G$ requires either $\Delta$ or $\Delta+1$ colors, where $\Delta$ is the maximum vertex degree of $G$. Holyer proved that it is $\mathbf{NP}$-complete to decide whether $G$ is $\Delta$-edge-colorable even for cubic graphs. By introducing the concept of complex colored edges, we show that the color-exchange operation follows the same multiplication rules as quaternion. An initially $\Delta$-edge-colored graph $G$ allows variable-colored edges, which can be eliminated by color exchanges in a manner similar to variable eliminations in solving systems of linear equations. The problem is solved if all variables are eliminated and a properly $\Delta$-edge-colored graph is reached. For a randomly generated graph $G$, we prove that our algorithm returns a proper $\Delta$-edge-coloring with a probability of at least $1/2$ in $O(\Delta |V|^{|E|/5})$ time if $G$ is $\Delta$-edge-colorable. Otherwise, the algorithm halts in polynomial time and signals the impossibility of a solution, meaning that the chromatic index of $G$ probably equals $\Delta+1$.

The $\Delta$-edge-coloring problem of bipartite graphs is completely solved. The best known algorithm for finding a proper $\Delta$-edge-coloring of a bipartite graph runs in time $O(|E| \log \Delta)$. The running time of our algorithm for bipartite graphs is on the order of $O(|E| \log |V|)$. As for non-bipartite graphs, the only known result is 3-edge-coloring of cubic planar graphs. As Tait proved that the 3-edge-coloring problem of bridgeless cubic planar graphs is equivalent to the four color map problem, the newly improved proof of four-color theorem actually can give rise to a quadratic algorithm for finding proper 3-edge-coloring.
of cubic planar graphs. Thus, our approach is the first randomized algorithm for finding $\Delta$-edge-coloring of general graphs.

Animations of the edge-coloring algorithms proposed in this work are posted at http://www.youtube.com/watch?v=KMnj4UMYl7k and http://v.youku.com/v_show/id_XMjJU3ODE0MTQ4.html.

This is a joint work with Yujie Wan and Hao Guan.