Spherical Designs and Universally Optimal Codes: a Survey

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Abstract

We explain the efforts of mathematicians to try to find good finite point sets on the sphere. What is good is a part of the problem. The concept of spherical t-design was introduced by Delsarte-Goethals-Seidel [3]. Spherical t-designs are finite sets on the sphere which can replace the sphere for the calculations of the integrals on the sphere of any polynomials of degree at most t. So, they are good finite sets approximating the whole sphere.

The concept of universally optimal code was introduced by Cohn-Kumar in [2]. Universally optimal codes are the energy minimizing finite sets on the sphere (among all finite sets of the same cardinality) for all reasonably good potential functions. They are rare to exist, but finding and classifying them are very interesting problems not only in mathematics but also in physics and other sciences.

In this talk, we review the recent developments on the studies of spherical t-designs and universally optimal codes, and the connections between them, emphasizing the viewpoint of algebraic combinatorics on spheres, following our survey article [1].

