Densely embedded subgraphs

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In Chapter 10 of my book [1] a classification project for a class of locally projective graphs is outlined. An important role in realization of this project is played by so-called Densely Embedded Subgraphs. Instead of presenting here the definitions from [1] we present the classical example. Let \((V, f, q)\) be a triple where \(V\) is a \(GF(2)\)-vector space of even dimension, \(f\) is a non-singular symplectic form on \(V\) and \(q\) is a quadratic form on \(V\) of maximal Witt index, whose associated bilinear form is \(f\). Then the graph on maximal and premaximal subspaces of \(V\) totally isotropic with respect to \(f\) is a locally projective graph and the subgraph formed by the subspaces which are totally isotropic with respect to \(q\) is densely embedded. This construction reflects the embedding

\[ O_{2m}^+(2) < Sp_{2m}(2). \]

In the non-classical part the following embedding of sporadic simple groups are realized through densely embedded subgraphs:

\[ M_{22} < M_{24}, \quad Co_2 < Co_1, \quad BM < M. \]

References