On the book 'Discrete Mathematics in Statistical Physics'

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Each chapter has some basic material, and in the end some more advanced material which is more briefly sketched.

Connectivity and Flows, Cycle space, Cut space

Factors, matchings, dimers.

Planar graphs

Tree-width and excluded minors

A brief introduction to: Graph colorings, random graphs, Ramsey theory, Regularity Lemma

Some open problems: matching preservers for non-bipartite graphs
Higher genus embeddings of a given graph.

Minimum spanning tree (greedy algorithm)
Tree isomorphism (How much information is contained in basic partition functions)
Tree enumeration, electric networks

A brief introduction to: Random walks

Some open problems: Stanley's isomorphism conjecture

3. Matroids, Geometric representations of graphs.

Matroids form a basis of discrete optimisation:
systems where rank is defined,
submodularity, on-line auctions, algorithmic game theory
greedy algorithm,
polyhedral methods, approximation algorithms ....

Matroids connect graph theory, linear algebra and optimization.

Chapter on Geometric representations of graphs: a bit advanced, important...

4. Game of dualities.

This topic is encountered in the theory of Kasteleyn orientations.

Geometric duality and matroidal duality
Van der Waerden theorem and Mac Williams theorem

A brief introduction to: Phase transitions, Yang-Baxter equation

Some open problems: strongly polynomial algorithm for MAX-CUT for toroidal square grids.

5. Graph functions, Knot theory.

This chapter on Graph polynomials connects graph polynomials and the discrete Ihara-Selberg functions.
These topics appear also in the knot theory.

These chapters are a bit advanced.
Some open problems: non-commutative formulas.

Mostly on the theory of Kasteleyn orientations: was introduced in my previous lecture.