

2020 年群与代数表示前沿进展 与展望研讨会

会 议 手 册



上海交通大学数学科学学院

2020年6月6-7日

2020 年群与代数表示前沿进展与展望研讨会

2020 年 6 月 6-7 日

由张继平院士发起,以北京大学、首都师范大学、华中师范大学、上海交通大学代数方向研究生为基础的暑期代数学研讨班和相应学术会议,自 2017 年已连续举办 3 年,得到南方科技大学、三峡数学中心、河北师范大学的大力支持;同时也在首师大和华中师大举办过群与代数表示前沿进展与展望研讨会.这些活动通过系列先进课程和前沿报告,旨在拓宽和加强研究生的现代数学基础,培养和提高他们的数学视野和能力;同时增进同行间的学术交流.

本次会议是上述活动的延续,得到上海交大数学科学学院和吴文俊数学中心的支持.会议邀请到 16 位著名代数学家和青年代数学家作特邀报告,涉及代数学前沿研究广泛的主题:

第 1 天上午的特邀报告涉及顶点算子代数,矩阵代数,拟遗传代数,和导出等价.

第 1 天下午的特邀报告均联系群表示,涉及有限单群理论,特征标理论的推广, Brauer 图,和 Alperin 权猜想.

第 2 天上午的特邀报告以范畴化方法研究 Hall 代数, Lie 代数和结合代数的表示.

第 2 天下午的特邀报告以同调代数为主题,包括奇点,同调猜想, Gorenstein 投射模,和 Calabi-Yau 代数.

由于疫情的限制,本次会议以 Zoom 视频会议的方式进行:

6 月 6 日 会议号 99009724426 会议密码: 495016

6 月 7 日 会议号 96853219890 会议密码: 749148

温馨提示:

- zoom 客户端 5.0 或以上版本才能使用: 下载地址: <https://zoom.us/download>
- 为保证会议正常运行,北京时间早上 7:15 起开放会议室,以备调式.
- 每场报告 50 分钟. 由于视频准备及研究生参会,请各位专家报告最好不超过 48 分钟.

• **预案** 如果 6 月 6 日会议开始前 15 分钟 Zoom 还不能正常运作 (这个概率小于 1%), 则会议立即移至腾讯会议 <https://meeting.tencent.com/s/pdy4ltSyD52m> 会议号 791 736 789

如果 6 月 7 日会议开始前 15 分钟 Zoom 还不能正常运作 (这个概率小于 1%), 则会议立即移至腾讯会议 <https://meeting.tencent.com/s/1dRWsdcSGjs4> 会议号 319 722 530

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2020年群与代数表示前沿进展与展望研讨会程序册

6月6日 会议号 99009724426 会议密码: 495016

时间	报告	主持人
8:00-8:30	开幕式	张继平
8:30-9:20	董崇英 (University of California, Santa Cruz, USA): Orbifold theory and modular extensions	
9:25-10:15	惠昌常 (首都师范大学): Invariant matrix algebras	
10:25-11:15	方明 (中科院数学所): Modules filtered by standard modules over quasi-hereditary algebras	
11:20-12:10	陈小伍 (中国科学技术大学): The conjecture of standard derived equivalences	
12:15-14:00	午休	
14:00-14:50	李才恒 (南方科技大学): Exact factorizations of almost simple groups	惠昌常
14:55-15:45	王立中 (北京大学): On the generalization of Frobenius theory of characters	
15:55-16:45	刘玉明 (北京师范大学): When is an associated graded algebra of a Brauer graph algebra representation-finite	
16:50-17:40	周远扬 (华中师范大学): Alperin Weight Conjecture for 2-blocks with abelian defect groups	

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6 月 7 日 会议号 96853219890 会议密码: 749148

时间	报告	主持人
8:30–9:20	林宗柱 (Kansas State University, USA): Partial categories, Hall algebras, and Grothendieck rings	周远扬
9:25–10:15	赵开明 (Wilfrid Laurier University, Waterloo, Canada): Actions of monoidal categories	
10:25–11:15	高楠 (上海大学): A functorial approach to monomorphism categories for species	
11:20–12:10	陈红星 (首都师范大学): Applications of derived decompositions of abelian categories	
12:15–14:00	午休	
14:00–14:50	叶郁 (中国科学技术大学): Pre-resolutions of noncommutative isolated singularities	姜翠波
14:55–15:45	胡维 (北京师范大学): Singular equivalences and homological conjectures	
15:55–16:45	黄兆泳 (南京大学): Applications of the transpose in Gorenstein homological theory	
16:50–17:40	吴泉水 (复旦大学): Homological (co)determinants and their applications	

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会议特邀报告摘要

Applications of derived decompositions of abelian categories

陈红星 (首都师范大学 chx19830818@163.com)

Derived decompositions of abelian categories are introduced in internal terms of abelian subcategories. They are used to construct semi-orthogonal decompositions (or in other terminology, Bousfield localizations, or hereditary torsion pairs) in derived categories of abelian categories. A sufficient condition is given for abelian categories to have derived decompositions. It is also necessary if abelian categories have enough projectives and injectives. Derived decompositions of abelian categories can be applied to construct complete cotorsion pairs in abelian categories, especially in the module categories of endomorphism rings of infinitely generated tilting modules over arbitrary rings, and also to establish inequalities of global dimensions of abelian categories.

This is a joint work with Changchang Xi.

The conjecture of standard derived equivalences

陈小伍 (中国科学技术大学 xwchen@mail.ustc.edu.cn)

In 1991, Jeremy Rickard proposed the following well-known conjecture: any derived equivalence between module categories is standard, that is, given by the derived tensor product of a two-sided tilting complex. We will report a series of papers devoted to this conjecture.

Orbifold theory and modular extensions

董崇英 (University of California, Santa Cruz, USA dong@ucsc.edu)

Orbifold theory studies a vertex operator algebra V under the action of a finite automorphism group G . The main objective is to understand the module category of fixed point vertex operator subalgebra V^G . We show that the module category of V^G can be understood in terms of the third cohomology group of G with coefficients in the unit circle if V is a nice vertex operator algebra. The idea is to establish a connection between the V^G -module category and modular extensions of G -module category. On the other hand, the modular extensions of G -module categories have been classified using the twisted Drinfeld quantum doubles of G in category theory. This talk will explain how to use the results on modular extensions by Drinfeld-Gelaki-Nikshych-Ostrik and Lan-Kong-Wen to study the module category of V^G . This is a joint work with Richard Ng and Li Ren.

Modules filtered by standard modules over quasi-hereditary algebras

方明 (中国科学院数学研究所 fming@amss.ac.cn)

The radical and socle filtrations are the most fundamental, but also the most difficult to understand the structure of a module in general. Given a module filtered by standard modules over a quasi-hereditary algebra, we replace simple modules, the key in the construction of radical and socle filtrations, by standard modules, to define Δ -radical filtration and Δ -socle filtration. We study some properties of the new filtration, like the Jordan-Holder type theorem on "composition factors". New filtration is more or less motivated by the previous work of Scott, Bowmann-Martin, and Hazi on various filtrations for modules over a quasi-hereditary algebra. It is "coarse" than the radical and socle filtrations, and therefore has chances to be described easily, especially for projective modules over Schur algebras. This is an ongoing joint work with Jun Hu.

A functorial approach to monomorphism categories for species

高楠 (上海大学 nangao@shu.edu.cn)

We introduce a very general extension of the monomorphism category as studied by Ringel and Schmidmeier which also covers generalised species over locally bounded quivers. We prove that analogues of the kernel and cokernel functor send almost split sequences over the path algebra and the preprojective algebra to split or almost split sequences in the monomorphism category. Despite of its generality, our monomorphism categories still allow for explicit computations as in the case of Ringel and Schmidmeier. This is based on the joint work with Julian Külshammer, Sondre Kvamme and Chrysostomos Psaroudakis.

Singular equivalences and homological conjectures

胡维 (北京师范大学 huwei@bnu.edu.cn)

Derived equivalences preserve several important homological conjectures such as Finitistic Dimension Conjecture and (Generalized) Auslander-Reiten Conjecture. The situation for singular equivalences seems to be unknown. In this talk, we will show that in certain cases singular equivalences do preserve these conjectures. This is based on joint works with Shengyong Pan; Yiping Chen, Yongyun Qin and Ren Wang.

Applications of the Transpose in Gorenstein Homological Theory

黄兆泳 (南京大学 huangzy@nju.edu.cn)

In this talk, I will review the origin and some progress of Gorenstein homological theory, and emphatically review some applications of the Auslander transpose in this theory.

Exact factorizations of almost simple groups

李才恒 (南方科技大学 lich@sustc.edu.cn)

Classifying exact factorizations of finite almost simple groups is one of fundamental problems in finite simple group theory, which was initiated by Ito's work for 2-dimensional linear groups in 1950's and the work of Wiegold and Williamson in 1980 for alternating and symmetric groups. In this talk, I will report on a solution of this long-standing problem. In order to accomplish this work, we prove that each factorization of an almost simple group has at least one factor to be almost maximal, which indicates the structure of finite simple groups is 'tight'. This is joint work with WANG Lei and XIA Binzhou.

Partial categories, Hall algebras, and Grothendieck rings

林宗柱 (Kansas State University, USA zlin@ksu.edu)

Alperin studied the fusion properties of subgroups of a Sylow p -subgroup of finite group in 1967, which lead to later development of fusion systems. Fusion system is regarded as a local group theory. In the attempt to characterize fusion systems in terms of homotopy theory, the concept of localities and partial groups were introduced. Around the same time Kriz and May also introduced partial algebras in an attempt to formalize the intersection multiplication of motives. It turns out that both concepts are pretty much the same in categorical setting, where multiplication needed not to be defined as they should. To do so one has to define the concepts of domains in both settings. The concept of partial categories can be defined in a similar fashion. Partial category can be used as a framework to bridge several different mathematical structures. In this talk I will focus on realizing Hall algebras of an abelian category as the Grothendieck ring of a partial category attached to the abelian category. This is an attempt to investigate the relations between Hall algebras and cohomology theory of simplicial sets.

When is an associated graded algebra of a Brauer graph algebra representation-finite

刘玉明 (北京师范大学 ymliu@bnu.edu.cn)

In representation theory of finite dimensional algebras, the graded algebras associated with the radical filtration often play an important role. Cline, Parshall and Scott pointed out that if A is a quasi-hereditary algebra with some condition, then the associated graded algebra $\text{gr}(A)$ is isomorphic to the double homological dual of A . Much less is known for both the ring theoretical properties and the representation theory of $\text{gr}(A)$ in other cases, for example if A has infinite global dimension.

To study the relationship between A and $\text{gr}(A)$, we choose A to be a Brauer graph algebra. Brauer graph algebras is a good class of algebras for our aim: on the one hand they are closely related to the modular representation theory of finite groups; on the other hand, they coincide with symmetric special biserial algebras. There has been a lot of work on Brauer graph algebras and their representation theory.

Given a Brauer graph algebra A associated with a Brauer graph G . In this talk, we give a criterion for the associated graded algebra $\text{gr}(A)$ to be representation-finite in terms of the graded degrees of vertices in G . Moreover, when $\text{gr}(A)$ is representation-finite, we give the precise relationship between the AR-quiver of A and the AR-quiver of $\text{gr}(A)$. We hope that our study could provide hints for researchers on what could be done on the topic of associated graded algebras.

This is a joint work with Jing Guo.

On the generalization of Frobenius theory of characters

王立中 (北京大学 lwang@math.pku.edu.cn)

In this talk, we will apply original method of Frobenius to a general setting. These generalizations will lead to the decomposition of group determinant in any subfield of complex field C and the notion of K -simple groups with respect to a field K . A more general formulation of Isaacs-Navarro-MacKay conjecture is also included as an application. This is a joint work with Prof. Jiping Zhang (张继平).

Homological (co)determinants and their applications

吴泉水 (复旦大学 qswu@fudan.edu.cn)

In the talk I will start from a short survey of Homological (co)determinants, then I will introduce some recent applications to the skew Calabi-Yau properties of some algebra constructions.

Invariant matrix algebras

惠昌常 (首都师范大学 xicc@cnu.edu.cn)

Given a subgroup G of the symmetric group Σ_n and an algebra R , the G -invariant matrix algebra $S_n(G, R)$ of order n over R consists of all n by n matrices over R that are invariant under the action of G . We investigate algebraic structures of this algebra and its relations to the full n by n matrix algebra $M_n(R)$ over R . Among other things, we will show when invariant matrix algebras are Morita equivalent, and when $M_n(R)$ is a Frobenius extension of $S_n(\sigma, R)$. For example, if R is a local ring, then $S_n(\sigma, R)$ and $S_{n+1}(\sigma^+, R)$ are Morita equivalent if and only if the quadratic form $\sum_{j=1}^s \lambda_j x_j y_j$ has a root in R , where σ^+ is build from σ by adding a cycle of length 1. The results in this talk generalise some results in [C. C. Xi and S. J. Yin, Cellularity of centrosymmetric matrix algebras and Frobenius extensions, Linear Algebra Appl. 590 (2020) 317-329].

Pre-resolutions of noncommutative isolated singularities

叶郁 (中国科学技术大学 yeyu@ustc.edu.cn)

We introduce the notion of right pre-resolutions (quasi-resolutions) for noncommutative isolated singularities, which is a weaker version of quasi-resolutions introduced by Qin-Wang-Zhang. We prove that two quasi-resolutions for a graded algebra with right injective dimension 2 are always Morita equivalent. We show that if a noncommutative quadric hypersurface is a noncommutative isolated singularity, then it admits a right pre-resolution. Moreover, we provide a method to verify whether a noncommutative quadric hypersurface is an isolated singularity. This is based on a joint work with Jiwei He.

Actions of monoidal categories

赵开明 (Wilfrid Laurier University, Waterloo, Canada kzhaow@wlu.ca)

Using crossed homomorphisms, we show that the category of weak representations (resp. admissible representations) of Lie-Rinehart algebras (resp. Leibniz pairs) is a left module category over the monoidal category of representations of Lie algebras. In particular, the corresponding bifunctor which we call the action of monoidal categories is established to give new weak representations (resp. admissible representations) of Lie-Rinehart algebras (resp. Leibniz pairs). This generalizes and unifies various existing constructions of representations of many Lie algebras by using this new bifunctor. This is a joint work with Y. Pei, Y. Sheng, R. Tang.

Alperin Weight Conjecture for 2-blocks with abelian defect groups

周远扬 (华中师范大学 zhouyuanyang@mail.ccnu.edu.cn)

In this talk, we prove that Alperin Weight Conjecture holds for 2-blocks with abelian defect groups. The proof depends on the classification of finite simple groups.