

# Variational Inequalities and Free Boundary Problems

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June 29, 2017

## Abstract

This mini-course introduces some basic theory on free boundary problems and its associated variational inequality problems. As an example, an American put problem will be thoroughly investigated. An outline of the course is as follows:

### Chapter 1 Variational Inequality

1. Obstacle Problem

Introduce the problem, the definition of variational solution, and the definition of viscosity solution.

2. A variational inequality arising from American put option

Present a brief derivation of the model. A numerical scheme is illustrated.

3. The Penalty Method.

Introduce the classical penalty method, based on the model problem of the variational inequality arising from the American put option.

### Chapter 2 Stefan Problem

1. Stefan problem and variational inequality

Briefly describe the classical Stefan problem. Attention is focus on the derivation of the Stefan problem from an variational inequality.

2. Well-posedness of the Stefan problem

Taking the free boundary problem arising from the American put option as an example, the well-posedness theory for the one-space dimensional Stefan problem is fully explained.

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3. Regularity of the free boundary

Present a standard theory, especially the boot strap argument, for the regularity of the free boundary.

4. Integral formulations

For theoretical as well as numerical analysis, integral formulations are derived for the free boundary problem arising from American put option.

Chapter 3 Convexity of free boundary for Stefan problem

In this chapter we present a technique used to establish the convexity of free boundary.

1. Lap number and maximum principle

2. Convexity of free boundary in American put option

3. From Stefan problem back to variational Inequality