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ABSTRACTS
Derived equivalences and Grothendieck constructions of oplax functors from a small category to the 2-cagery of $k$-categories

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We fix a commutative ring $k$ and a small category $I$, and denote by $\mathbf{k}$-$\text{Cat}$ the 2-category of $k$-categories. By regarding a group $G$ as a category $G$ with a single object $*$ and $G(\ast, \ast) := G$, a $G$-action on a $k$-category $C$ is nothing but a functor $G \to \mathbf{k}$-$\text{Cat}$ with $\ast \to C$; and a “pseudo-action” of $G$ on $C$, i.e., a group homomorphism $G \to \text{Auteq}(C) \cong \langle \text{the group of autoequivalences of } C \text{ modulo natural isomorphisms} \rangle$ is justified as a pseudo-functor $G \to \mathbf{k}$-$\text{Cat}$ with $\ast \to C$.

Generalizing these we consider an oplax functor $X : I \to \mathbf{k}$-$\text{Cat}$, even for which we can construct a $k$-category $\text{Gr}(X)$ as a $k$-linear version of the Grothendieck construction [2, Exposé VI §8] ($\text{Gr}(X)$ coincides with the orbit category $C/G$ when $X$ is a functor $G \to \mathbf{k}$-$\text{Cat}$ with $\ast \to C$ as above). The class of oplax functors $I \to \mathbf{k}$-$\text{Cat}$ forms a 2-category $\text{Oplax}(I, \mathbf{k}$-$\text{Cat})$ as explained in [3]. We define its “module category” $\text{Mod}_X \in \text{Oplax}(I, \mathbf{k}$-$\text{Ab})$ and “derived category” $D(\text{Mod}_X) \in \text{Oplax}(I, \mathbf{k}$-$\text{Tri})$, where $\mathbf{k}$-$\text{Ab}$ (resp. $\mathbf{k}$-$\text{Tri}$) is the 2-category of abelian (resp. triangulated) $k$-categories. For $X$, $X'$ in $\text{Oplax}(I, \mathbf{k}$-$\text{Cat})$ they are defined to be derived equivalent if $D(\text{Mod}_X)$ and $D(\text{Mod}_{X'})$ are equivalent in the 2-category $\text{Oplax}(I, \mathbf{k}$-$\text{Tri})$. An oplax functor $X$ is called $k$-projective if $X(i)(x, y)$ is a projective $k$-module for each $i \in I$ and $x, y \in X(i)$. Note that all oplax functors are $k$-projective when $k$ is a field. Our main result is the following:

**Theorem.** Let $X$ and $X'$ be oplax functors $I \to \mathbf{k}$-$\text{Cat}$ and consider the following conditions.

1. $X$ and $X'$ are derived equivalent;
2. There exists a “tilting oplax subfunctor” $T$ for $X$ such that $T$ and $X'$ are equivalent in $\text{Oplax}(I, \mathbf{k}$-$\text{Cat})$;
3. $\text{Gr}(X)$ and $\text{Gr}(X')$ are derived equivalent.

Then

(a) $(1) \Rightarrow (2)$;
(b) $(2) \Rightarrow (3)$; and
(c) If $X'$ is $k$-projective, then $(2) \Rightarrow (1)$.

**Remark.** We note the following.

(i) The statements (a) and (c) give a generalization of the Morita type theorem characterizing derived equivalences of categories by Rickard and Keller in our setting.
(ii) The statement (b) gives a generalization of [1, Theorem 4.11].
(iii) By (a) and (b), we have $(1) \Rightarrow (3)$. As an easy application, this gives a unified proof of the fact that if $A$ and $A'$ are derived equivalent $k$-algebras, then so are their quiver algebras $AQ$, $A'Q$, incidence algebras $AS$, $A'S$ and semigroup algebras $AG$, $A'G$ for all quivers $Q$, posets $S$ and semigroups $G$.

**References**


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Realizing cohomology modules over commutative local rings

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Given a commutative noetherian ring $R$ with unique maximal ideal $m$ and residue field $k = R/m$, which graded modules over the Yoneda algebra $Ext^*_R(k, k)$ are isomorphic to $Ext^*_R(M, k)$ for some finitely generated $R$-module $M$? Recent joint results with David Jorgensen identify large classes of such modules over several types of local rings, including complete intersections and Golod rings.

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Descent from subgroups to ambient group

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The category of representations of a subgroup $H$ in an ambient group $G$ is equivalent to the category of modules inside the category of representations of $G$, with respect to a suitable monad or even with respect to a ring object when the index of $H$ in $G$ is finite. This allows the transposition of the (classical) techniques of descent to the question of extending representations from a subgroup $H$ to the group $G$.

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Projective dimension of modules over cluster-tilted algebras

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We study the projective dimension of modules over a cluster-tilted algebra $\text{End}_C(T)$ where $T$ is a tilting object in a cluster category $C$. It is well-known that all modules are of the form $\text{Hom}(T, M)$ for some object $M$ in $C$, and since $\text{End}(T)$ is Gorenstein of dimension 1, the projective dimension of $\text{Hom}(T, M)$ is either zero, one or infinity.

We consider the ideal $I_M$ of $\text{End}_C(T[1])$ given by all endomorphisms that factor through $M$, and show that the $\text{End}(T)$—module $\text{Hom}(T, M)$ has infinite projective dimension precisely when $I_M$ is non-zero. Examples indicate that the objects $M$ of $C$ such that $I_M \neq 0$ lie on hammocks in $C$.

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Representations of finite dimensional pointed Hopf algebras over \(\mathbb{Z}_n\)

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We study the representations of the new finite dimensional pointed Hopf algebras over the cyclic group of order \(n\) in positive characteristic. It is proved that these Hopf algebras are symmetric algebras. The simple modules and the indecomposable modules over these Hopf algebras are determined. It is also shown that these Hopf algebras are of wild representation type.

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Singularity categories and Leavitt path algebras

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We study the singularity category of a finite dimensional algebra with radical square zero. It is closely related to Leavitt path algebras. We also discuss a singular equivalence between certain algebras given by bimodules. This is related, via the work of Smith, to strong shift equivalence in symbolic dynamic systems and Morita equivalence of Leavitt path algebras.

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A realization of elliptic Lie algebras of type \(F_4^{(2,2)}\) by the Ringel-Hall approach

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In this talk, we will discuss the relations between an elliptic Lie algebra \(\mathfrak{g}\) of type \(F_4^{(2,2)}\) and the \(F\)-fixed point algebra \(A^F\) of a tubular algebra \(A\) of type \(\mathbb{T}(3,3,3)\) under a Frobenius morphism \(F\). Using the explicit structure of the root category of the \(F\)-fixed point algebra \(A^F\), we prove that the elliptic Lie algebra \(\mathfrak{g}\) of type \(F_4^{(2,2)}\) is isomorphic to the Ringel-Hall Lie algebra of the root category of the \(F\)-fixed point algebra \(A^F\). This is joint work with Yanan Lin.

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Fundamental group of Schurian categories and the Hurewicz isomorphism

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Joint work with M. J. Redondo and A. Solotar (to appear in Documenta Mathematica). We will begin by recalling the Hurewicz morphism from the first Hochschild cohomology vector space of a linear category over a field to the abelian characters of the group of a Galois covering (corresponding to the abelianization in algebraic topology) and conditions for this map for being injective. Then to any Schurian category we attach a CW-complex and we prove that its fundamental group is isomorphic to the intrinsic fundamental group “à la Grothendieck” of the linear category. We consider Schurian categories with no restrictions in the set of objects nor requiring an admissible presentation of it. This way we extend previous results by J.C. Bustamante obtained for a given presentation of a finite number of objects Schurian category. Moreover this is compatible with the fact that the intrinsic fundamental group is functorial with respect to full subcategories. Finally the Hurewicz morphism is an isomorphism for Schurian categories.

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Loewy lengths of tensor products of $kD_2l$-modules

Erik Darpo
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Let $G = D_{2l}$ be a dihedral 2-group, and $k$ a field of characteristic 2. I shall present a formula for the Loewy length of the tensor product of two $kG$-modules. In particular, this gives an explicit criterion for when such a tensor product contains a non-zero projective direct summand.

The method used to obtain the result is to first find a formula for the length of a tensor product of two uniserial modules, and then show that the general problem is reducible to this special case.

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Representations of affine quantum Schur algebras

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We classify irreducible representations of affine quantum Schur algebras and indicate the bridging role played by affine quantum Schur algebras between representations of quantum affine $gl_n$ and those of affine Hecke algebras of type $A$.

This is joint work with Jie Du and Qiang Fu.

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Lie algebras for finite dimensional algebras
via lattice vertex operator algebras

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We study Lie algebras associated to finite dimensional algebra with finite global dimension via lattice vertex operator algebras. Our main results involve two classes of finite dimensional algebras: 1) algebras with weakly positive Tits form; 2) canonical algebras. For any weakly positive form \( q \), we introduce a Lie algebra \( \mathfrak{g}_q \) via lattice vertex operator algebra associated to the even lattice defined by \( q \). If \( q \) is the Tits form of a representation-directed algebra \( A \), \( \mathfrak{g}_q \) turns out to be the Ringel Hall Lie subalgebra \( \mathcal{LH}(A) \) introduced by Ringel. For a canonical algebra \( A \), we consider the Lie algebra \( \mathfrak{g}(A) \) generated by the vertex operators associated to the simple modules and their suspension. We establish a connection between \( \mathfrak{g}(A) \) and the universal central extension of loop algebra for some Kac-Moody algebra. This is a joint work with Liangang Peng.

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Homological conjectures and stable equivalences for algebras of finite Cohen-Macaulay type

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We study homological conjectures for finite-dimensional algebras of finite Cohen-Macaulay type using the language of Gorenstein derived category. The corresponding version of Happel’s results are obtained. We also show that \( eAe \) is of finite Cohen-Macaulay type for a finite-dimensional bimodule Calabi-Yau algebra \( A \) with \( e \) an idempotent. We show that if their relative Auslander algebras are stably equivalent for two artin algebras of finite Cohen-Macaulay type, then their relative stable module categories are equivalent.

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Returning arrows for graded self-injective Algebras

JIN YUN GUO
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The phenomenon of producing returning arrows in the quiver are observed in recent construction of as cluster tilting objects. In this talk, we show how returning arrows appear in the McKay quivers and trivial extensions of graded self-injective algebras, and we also discuss their relationship with numerate invariants such as complexities.

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Rocollements and Hochschild theory

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I will talk about the relations between recollements of derived categories of algebras and Hochschild dimension, Hochschild homology, and Hochschild cohomology of algebras.

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Strong global dimension

DIETER HAPPEL
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This is a report on joint work with Dan Zacharia

Let $\Lambda$ be a finite dimensional algebra over a field $k$. Over twenty years ago Ringel proposed to consider for a finite dimensional algebra the notion of strong global dimension. Roughly it measures the length of indecomposable complexes whose stalks are projective $\Lambda$-modules. Recall that $\Lambda$ is said to be piecewise hereditary, if there exists a hereditary, abelian category $\mathcal{H}$ such that the bounded derived categories $D^b(\Lambda)$ and $D^b(\mathcal{H})$ are equivalent as triangulated categories. It is known that the algebra $\Lambda$ is piecewise hereditary if and only if the strong global dimension of $\Lambda$ is finite. We will first recall a few facts on piecewise hereditary algebras, but the main focus of the talk is the actual behavior of the strong global dimension. We will see how it behaves with respect to tilting and one point extensions. We will also give upper bounds depending on the chosen derived equivalence. A good lower bound seems to be unknown. If time permits we will also present some structural results for indecomposable complexes of projective $\Lambda$-modules for a piecewise hereditary algebra $\Lambda$.

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Representations of super Yang-Mills algebras via the Dixmier map for nilpotent super Lie algebras

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This talk is concerned with the existence of a Dixmier map for nilpotent super Lie algebras and its applications to the representation theory of super Yang-Mills algebras. More precisely, we shall state results concerning the Kirillov orbit method à la Dixmier for nilpotent super Lie algebras, i.e. that the usual Dixmier map for nilpotent Lie algebras can be naturally extended to the context of nilpotent super Lie algebras. Moreover, our construction of the previous map is explicit, and more or less parallel to the one for Lie algebras, a major difference with a previous approach. One key fact in the construction is the existence of polarizations for (solvable) super Lie algebras, generalizing the concept in the non super case. As a corollary of the previous description, we obtain that the quotient of the enveloping algebra of a nilpotent super Lie algebra by a maximal ideal is isomorphic to the tensor product of a Clifford algebra and a Weyl algebra, and we determine explicitly the indices of both of them, All of these results can be used to study the representation theory of super algebras related to the super Yang-Mills theory of interest in physics.

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The construction of proper resolutions

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It is shown how to construct a (strongly) proper resolution of one term in a short exact sequence from that of the other two terms, and then the construction is applied to the study of the properties of modules satisfying the Auslander condition and relative homological theory.

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$n$-representation infinite algebras

Osamu Iyama
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This is a joint work with Martin Herschend and Steffen Oppermann. From viewpoint of higher dimensional AR theory, there are two important classes of finite dimensional algebras with global dimension $n$. One is a class of $n$-representation finite algebras, and the other is a class of $n$-representation infinite algebras. For the classical case $n = 1$, they are path algebras of Dynkin quivers and non-Dynkin quivers respectively. Both classes are characterized in terms of the corresponding $(n + 1)$-preprojective algebras. In this lecture I will discuss $n$-representation infinite algebras. Similar to the classical case $n = 1$, there are three fundamental classes of modules, $n$-preprojective modules, $n$-preinjective modules and $n$-regular modules. We show that all of them have nice descriptions in terms of the corresponding $(n + 1)$-preprojective algebras. We conjecture that $n$-representation infinite algebras always have representation dimension at least $n + 2$. We prove this for ‘$n$-representation tame’ algebras as an application of our results.

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Cluster algebras, quantum dilogarithms and Calabi-Yau categories

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In this talk, we will review the construction of quantum cluster transformations via quantum dilogarithms due to Fock and Goncharov. We will link it to the study of cluster collections in 3-Calabi-Yau categories (following Kontsevich-Soibelman) and that of cluster-tilting objects in 2-Calabi-Yau categories. We will show how the suspension functors of these categories give rise to remarkable automorphisms of quantum cluster algebras.

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Symmetric powers, Brauer algebras and Schur algebras

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Classical Schur-Weyl duality relates Schur algebras of infinite general linear groups with finite symmetric groups via commuting actions on tensor space. Similarly, Schur algebras of symplectic or orthogonal groups are related with Brauer algebras. Now replace tensor space by a direct sum of tensor products of symmetric powers. For general linear groups, the endomorphism ring is the classical Schur algebra. For orthogonal and symplectic groups, however, the endomorphism ring is a new algebra, which at the same time plays the role of a Schur algebra for Brauer’s algebra. (This is joint work with Anne Henke.)

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Functors and morphisms determined by objects revisited

Henning Krause
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The concept of a functor or morphism determined by an object was introduced by Maurice Auslander in 1978 as an attempt to generalise previous results (joint with Idun Reiten) on the existence of almost split sequences. It seems that this fundamental contribution of Auslander is almost forgotten. So I will explain this circle of ideas and I will show that this concept is particularly useful for studying morphisms in triangulated categories.

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The two-flag problem for nilpotent operators, and weighted projective lines

Helmut Lenzing
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My talk is on joint work with D. Kussin and H. Meltzer. It deals with the problem to investigate the category of (graded) nilpotent operators, equipped with two finite flags of invariant subspaces. This problem is non-trivial even if the two flags degenerate to a single invariant subspace, a problem studied by C.M. Ringel and M. Schmidmeier in recent years. Actually the problem goes back — in the context of finite abelian groups — to G. Birkhoff in the 1930s. General aspects, have been recently studied by various authors, among them D. Simson and P. Zhang. A bit surprisingly, the graded two-flag problem is contained (in a certain sense) in the problem to classify indecomposable vector bundles on a weighted projective line of triple weight type. We show among other results that for fixed flag length and nilpotency degree the invariant subspace problem for nilpotent operators yields an almost Frobenius category whose attached stable category is triangulated and then triangle-equivalent to the stable category of vector bundles on an appropriate weighted projective line of triple weight type. We will discuss in some detail the tools from weighted projective lines entering the proof, and the reason why triple weight type is essential for the results.

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The derived minimal dg algebras

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Recently, Benson, Iyengar and Krause have proved that $K(InjG)$, the homotopy category of injective $kG$-modules for a finite group $G$ is stratified by the cohomology ring of $G$. Also they generalize Neeman’s classifying theorem on commutative rings to the formal commutative dg algebras. Motivated by their work, we call a dg algebra derived minimal if its derived category of dg-modules has no proper localizing subcategories. In this talk, I will show that how to construct some derived minimal dg algebras, by which we can describe the minimal localizing subcategories of $K(Inj(kZ/2Z \times Z/2Z))$ and the derived category $D(ModZ)$, respectively.

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The representation theory of an infinite quiver

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(joint with Raymundo Bautista and Charles Paquette)

The representation theory of a finite quiver has been extensively studied over the last four decades. On the other hand, in some recent development, one needs to deal with representations of an infinite quiver, for example, in the classification of noetherian hereditary abelian category with a Serre functor and in the study of the derived category of a finite dimensional algebra with radical squared zero.

Let $Q$ be a connected infinite quiver which is locally finite and interval-finite (that is, the number of paths between any two given vertices is finite). The main objective of this talk is to study the Auslander-Reiten theory in the category $\text{rep}^+(Q)$ of finitely presented representations of $Q$ over an arbitrary field. We first give some necessary and sufficient conditions for $\text{rep}^+(Q)$ to have (left, right) almost split sequences. This yields many interesting examples of Ext-finite hereditary abelian categories which have or do not have almost split sequences. In the most general context, we describe the shapes of the connected components of the Auslander-Reiten quiver $\Gamma_{\text{rep}^+(Q)}$, which consist of a unique preprojective component, from zero to infinitely many preinjective components, and four types of regular components. In case $Q$ is of infinite Dynkin type, we are able to characterize completely the indecomposable objects in $\text{rep}^+(Q)$ and show that $\Gamma_{\text{rep}^+(Q)}$ has at most four components. In case $Q$ is not of infinite Dynkin type, we show that $\Gamma_{\text{rep}^+(Q)}$ has always infinitely many regular components.

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Minimal Horseshoe Lemma and certain Koszulity

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It is well-known that Horseshoe Lemma is a basic tool in homological algebra. But unfortunately its “minimal” version fails in general. In this talk, we will give some necessary and sufficient conditions for the “minimal Horseshoe Lemma” to be true in terms of certain Koszul and quasi-Koszul objects. Moreover, some applications are also given.

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Quotient closed subcategories of quiver representations II

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This is the second part of a report on joint work with Idun Reiten and Hugh Thomas. We study subcategories of the category of representations of a quiver $Q$, which are closed under taking direct sums and quotients. It turns out that there is a bijection between the elements of the Coxeter group corresponding to $Q$, and the quotient closed subcategories missing only finitely many indecomposable representations, and that this bijection can be understood via preprojective algebras.

In this talk I will illustrate how one can use induction to establish the above result: We start by letting the unit of the Coxeter group correspond to the entire category of representations. For quotient closed subcategories missing some indecomposable representations, we use APR-tilting to reduce to a quotient closed subcategory missing fewer representations. Towards the end of my talk I will focus on the special features appearing if $Q$ is of Dynkin type. In this case we have a duality between ideals of the preprojective algebra and corresponding quotients. This immediately gives us a dual classification of subobject closed subcategories for Dynkin quivers.

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Exchange graphs and their applications

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We study the principal component of the oriented exchange graph of hearts in the finite-dimensional derived category $\mathcal{D}(\Gamma_N Q)$ of the Ginzburg algebra of degree $N$ associated to an acyclic quiver $Q$. We show that any such heart is induced from some heart in the bounded derived category $\mathcal{D}(Q)$ via some ‘immersion’ $F : \mathcal{D}(Q) \to \mathcal{D}(\Gamma_N Q)$. Further, we show that the quotient graph under the Seidel-Thomas braid group action is the exchange graph for $(N - 1)$-clusters, which can also be constructed as the cyclic completion of a subgraph of the exchange graph of $\mathcal{D}(Q)$.

If there is time, I could explain the example of exchange graph when $Q$ is type $A_2$ via Farey graph; or/and I could show the application to stability spaces and the topological realization of almost complete tilting objects.

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Quotient closed subcategories of quiver representations I

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This talk is the first part of a series of two lectures based on joint work with Steffen Oppermann and Hugh Thomas. The second part is given by Steffen Oppermann. For a quiver $Q$ we give a bijection between subcategories of representations of $Q$ closed under direct sums and quotients, missing only a finite number of indecomposable objects, and elements in the Coxeter group associated with $Q$.

In this talk we first give some background material on ideals, algebras and categories associated with elements in Coxeter groups. Then we discuss the main correspondence theorem, including giving examples for illustration. We also discuss a description of the subcategories of quiver representations in terms of ideals in the preprojective algebra associated with $Q$.

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Fibonacci numbers and representations of quivers

Claus Michael Ringel
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Looking at certain representations of the 3-Kronecker quiver and its universal cover, the 3-regular star with bipartite orientation, one obtains partition formulae for the Fibonacci numbers; these Fibonacci modules seem to provide a convenient categorification of the Fibonacci numbers. The basic information can be arranged in order to obtain two triangles which are quite similar to the Pascal triangle of the binomial coefficients, but in contrast to the additivity rule for the Pascal triangle, we now deal with additive functions for valued translation quivers. This concerns joint investigations with Ph. Fahr.

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How homomorphisms factor through Auslander-Reiten components

Markus Schmidmeier
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Given a homomorphism between modules over a finite dimensional algebra, and a connected component of the Auslander-Reiten quiver, it is natural to ask through which objects in the component the homomorphism factors.

We visualize in several cases of finite, tame and wild representation type, how certain particularly simple maps in the category of invariant subspaces of nilpotent linear operators factor through Auslander-Reiten components.

Extending to possibly decomposable middle terms, we show how operations on arc diagrams control the dimensions of the homomorphism spaces.

This is part of a joint project with Justyna Kosakowska from Torun University.

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Representation dimension and tilting

Luise Unger
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This is a report on joint work with Dieter Happel.

Let $\Lambda$ be a finite dimensional $k$-algebra over an algebraically closed field $k$. With rep.dim$\Lambda$ we denote the representation dimension of $\Lambda$. Let $T$ be a splitting tilting module over $\Lambda$ of projective dimension at most one and $\Gamma = \text{End}_\Lambda T$.

We indicate the proof that $\text{rep.dim}\Gamma \leq \text{rep.dim}\Lambda$ provided the representation dimension of $\Lambda$ is at most three. As a consequence it follows that the representation dimension of a piecewise hereditary algebra is at most three.

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Actions on Calabi-Yau algebras and their PBW-deformations

Quanshui Wu
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I will talk about actions on Koszul Calabi-Yau algebras, and show that the Calabi-Yau property is preserved if and only if the homological determinant of the action is trivial. I will also talk about the Poincare-Birkhoff-Witt deformation of Calabi-Yau algebras, and give a necessary and sufficient condition for the PBW-deformations of Koszul Calabi-Yau algebras to be Calabi-Yau. The work in this talk is joint with Zhu Can.

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Auslander-Reiten translations in monomorphism categories

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This is a joint work with Pu Zhang and Yuehui Zhang.

We generalize Ringel and Schmidmeier’s theory on the Auslander-Reiten translation of the submodule category $S_2(A)$ to the monomorphism category $S_n(A)$. As in the case of $n = 2$, $S_n(A)$ has Auslander-Reiten sequences, and the Auslander-Reiten translation $\tau_S$ of $S_n(A)$ can be explicitly formulated via $\tau$ of $A$-mod.

Furthermore, if $A$ is a selfinjective algebra, we study the periodicity of $\tau_S$ on the objects of $S_n(A)$, and the periodicity of the Serre functor $F_S$ on the objects of the stable monomorphism category $S_n(A)$.

In particular, $\tau_S^{2m(n+1)}X \cong X$ for $X \in S_n(A(m,t))$; and $F_S^{m(n+1)}X \cong X$ for $X \in S_n(A(m,t))$, where $A(m,t)$, $m \geq 1$, $t \geq 2$, are the selfinjective Nakayama algebras.

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A quantum analogue of generic bases for affine cluster algebras

FAN XU

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Generic bases for cluster algebras are analogous to Lusztig’s dual semi-canonical bases and have nice geometric properties. Recently, Geiss-Leclerc-Schorer proved that there exist geometric bases for acyclic cluster algebras. In this talk, I will explain my recent joint work with Ding Ming about the construction of a quantum version of affine generic bases.

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Derived categories of graded gentle one-cycle algebras

Dong Yang

This is a joint work with Martin Kalck.

The derived category of a finite-dimensional algebra $A$ is either discrete, or tame, or wild. Vossieck proved that $A$ is derived discrete if and only if $A$ is a gentle one-cycle algebra satisfying the clock condition. The Auslander–Reiten quiver of such a derived category is described in the work of Bobinski, Geiss and Skowronski. Those gentle one-cycle algebras which do not satisfy the clock condition are derived equivalent to an affine quiver of type $A$, as shown by Assem and Sknowronski.

We consider graded gentle one-cycle algebras. Their derived categories are either discrete or tame; and we describe the associated Auslander–Reiten quivers. The idea of the proof is to show that each graded gentle one-cycle algebra, as a dg algebra, is derived equivalent to a graded affine quiver of type $A$. On the way, we discuss the relation between the derived category of graded modules over a graded algebra and the derived category of the graded algebra considered as a dg algebra with trivial differential.

References


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Generalized $d$-Koszul modules

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This is a joint work with Ning Bian and Pu Zhang.

Higher Koszul structures are useful for example in the Artin-Shelter algebras, the Calabi-Yau algebras, and the Yang-Mills algebras.

Let $\Lambda$ be a $d$-Koszul algebra and $M$ a $d$-Koszul $\Lambda$-module. It was shown by E. L. Green et al. that the even Ext-algebra $E^{ev}(\Lambda)$ is a Koszul algebra and the even Ext-module $E^{ev}(M)$ is a Koszul $E^{ev}(\Lambda)$-module. This generalizes the corresponding result of J. Backelin and R. Fröberg on the local Koszul algebras. An open problem was raised by Green et al.: Is the odd Ext-module $E^{odd}(M)$ also a Koszul module over $E^{ev}(\Lambda)$?

We introduce the so-called generalized $d$-Koszul modules to affirmatively answer this problem. It turns out that generalized $d$-Koszul modules are quite natural. For example, the syzygies of a $d$-Koszul module are generalized $d$-Koszul up to shifts. Also for each $i$, $J^iM$ is a generalized $d$-Koszul module up to shift, where $M$ is a generalized $d$-Koszul module over a $d$-Koszul algebra, and $J$ is the graded Jacobson radical.

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Auslander-Reiten components with modules of finite complexity

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Let $R$ be a selfinjective finite dimensional algebra over an algebraically closed field. The complexity of a module measures the rate of growth of a minimal projective resolution. More precisely, if

$$P^\bullet: \cdots \to P^2 \xrightarrow{\delta_2} P^1 \xrightarrow{\delta_1} P^0 \xrightarrow{\delta_0} M \to 0$$

is a minimal projective resolution of a finitely generated $R$-module $M$, then we define the complexity of $M$ as

$$\text{cx } M = \inf \{ n \in \mathbb{N} \mid \dim P^i \leq ci^{n-1} \text{ for a positive number } c \in \mathbb{Q} \text{ and all } i \geq 0 \}$$

If no such $n$ exists, then we say that the complexity of $M$ is infinite. For instance, a module has complexity 0 if and only if it is projective, and its complexity is 1 if and only if it has a “bounded” projective resolution. I will talk about the possible shapes of the connected components of the Auslander-Reiten quiver containing modules of finite complexity.

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Monic representations and maximal Cohen-Macaulay modules

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This is a joint work with Xiuhua Luo. For a finite quiver \( Q \) and a \( k \)-algebra \( A \), we have the path algebra \( \Lambda \) of \( Q \) over \( A \). Then \( \Lambda \)-mod \( \cong \text{Rep}(Q, A) \), where an object in \( \text{Rep}(Q, A) \) is \( X = (X_i, X_\alpha, i \in Q_0, \alpha \in Q_1) \) with \( X_i \) an \( A \)-module and \( X_\alpha : X_{s(\alpha)} \to X_{t(\alpha)} \) an \( A \)-map. It is a monic representation if each \( X_\alpha \) is injective, and \[ \sum_{\alpha \in Q_1, t(\alpha)=i} \text{Im} X_\alpha = \bigoplus_{\alpha \in Q_1, t(\alpha)=i} \text{Im} X_\alpha, \forall i \in Q_0. \]

**Theorem.** If \( Q \) has no oriented cycles and \( A \) is finite-dimensional, then \( X \) is a maximal Cohen-Macaulay \( \Lambda \)-module if and only if \( X \) is a monic representation, and for each \( i \in Q_0 \), \( X_i \) and \( X_i/ \bigoplus_{\alpha \in Q_1, t(\alpha)=i} \text{Im} X_\alpha \) are maximal Cohen-Macaulay \( A \)-modules.

As a consequence, if \( A \) is self-injective, then a \( \Lambda \)-module \( X \) is maximal Cohen-Macaulay if and only if it is a monic representation. Thus, when in addition \( Q = \bullet \longrightarrow \bullet \), this goes to the submodule category, studied by C. M. Ringel, M. Schmidmeier, D. Simson, D. Kussin, H. Lenzing, H. Meltzer, X. W. Chen, and other authors.

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Global dimensions of endomorphism algebras of generator-cogenerators over \( m \)-replicated algebras

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This is a joint work with Hongbo Lv. Let \( A \) be a hereditary Artin algebra and \( A^{(m)} \) the \( m \)-replicated algebra of \( A \). Dlab and Ringel describe the global dimension of the endomorphism algebras of generator-cogenerators via the cardinalities of the Auslander-Reiten orbits. In this talk we show how to generalize their results to \( A^{(m)} \). Our main results are as follows.

**Theorem 1.** Let \( A^{(m)} \) be the \( m \)-replicated algebra of a representation-finite hereditary artin algebra \( A \) and \( d \) be an integer with \( d \geq 2 \). There exists an \( A^{(m)} \)-module \( M \) which is a generator-cogenerator with global dimension \( d \) if and only if there exists a \( \tau_{A^{(m)}} \)-orbit of cardinality at least \( d \).

**Theorem 2.** Let \( A^{(m)} \) be the \( m \)-replicated algebra of a representation-infinite hereditary artin algebra \( A \) and let \( d \) be either an integer with \( d \geq 3 \) or else the symbol \( \infty \). Then there exists an \( A^{(m)} \)-module \( M \) which is a generator-cogenerator with global dimension \( d \).

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Comparison morphisms and Hochschild cohomology

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We give a general recursive method to construct comparison morphisms between two projective resolutions of a module. As applications, for monomial algebras, we construct comparison morphisms between the reduced bar resolution and Bardzell’s minimal projective bimodule resolution, which enable us to give explicit formulae for the cup product and the Gerstenhaber Lie bracket over the Hochschild cohomology of a monomial algebra.

This talk is based on a joint work in progress with Jue Le.

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Mutation of torsion pairs and its geometric realization arising from marked surfaces

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My talk is on joint work with Bin Zhu.

By generalizing mutation of rigid subcategories, maximal rigid subcategories and cluster tilting subcategories, we introduce the notion of mutation of torsion pairs in triangulated categories. It is proved that the mutation of torsion pairs in triangulated categories are torsion pairs. As an application we classify all of torsion pairs in the cluster categories of marked surfaces and give a geometric realization of mutation of torsion pairs in this case.

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