Spectral radius and degree sequence of a graph

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Let $G$ be a simple connected graph of $n$ vertices and $m$ edges with degree sequence $d_1 \geq d_2 \geq \cdots \geq d_n$. The spectral radius $\rho(G)$ of $G$ is the largest eigenvalue of its adjacency matrix. The spectral radius of graphs has been studied by many authors. It is well-known that $\rho(G) \leq d_1$. In 1985 [1, Corollary 2.3], Brualdi and Hoffman proved that if $m \leq k(k-1)/2$ then

$$\rho(G) \leq k - 1. \quad (1)$$

In 1987 [2], Stanley improved (1) and showed that

$$\rho(G) \leq \frac{-1 + \sqrt{1 + 8m}}{2}. \quad (2)$$

In 2001 [3] Theorem 2.3], Hong et al. improved (2) and showed that

$$\rho(G) \leq \frac{d_n - 1 + \sqrt{(d_n + 1)^2 + 4(2m - nd_n)}}{2}. \quad (3)$$

In 2004 [4] Theorem 2.2], Jinlong Shu and Yarong Wu showed that

$$\rho(G) \leq \frac{d_\ell - 1 + \sqrt{(d_\ell + 1)^2 + 4(\ell - 1)(d_1 - d_\ell)}}{2} \quad (4)$$

for $1 \leq \ell \leq n$. Moreover, they showed in [4] Theorem 2.5] that if $p + q \geq d_1 + 1$ then (4) improves (3) where $p$ is the number of vertices with the largest degree $d_1$ and $q$ is the number of vertices with the second largest degree.

In this research, we present a sharp upper bound of $\rho(G)$ in terms of the degree sequence of $G$ which improves (4). We show that for $1 \leq \ell \leq n$,

$$\rho(G) \leq \phi_\ell = \frac{d_\ell - 1 + \sqrt{(d_\ell + 1)^2 + 4 \sum_{i=1}^{\ell-1} (d_i - d_\ell)}}{2},$$

with equality if and only if $G$ is regular or there exists $2 \leq t \leq \ell$ such that $d_1 = d_{\ell-1} = n-1$ and $d_t = d_n$. This result also improves [3] since $\phi_n$ is exactly the upper bound in [3].

This is a joint work with Professor Chih-wen Weng (weng@math.nctu.edu.tw).

References


